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FLOW STABILIZATION IN THE MULTILAYER VISCOELASTIC TUBES AT NO DISPLACEMENT BOUNDARY CONDITIONS

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Flow instability in the distensible tubes is studied in application to the blood flow and wave propagation in the blood vessels as multi-layer viscoelastic tubes. The axisymmetric disturbance and the no displacement boundary conditions corresponding to the deep vessels attached to the surrounding tissues are considered. It was shown the unstable fluid-based mode can be stabilized by certain choice of the viscosity and elastic modules of the layers at wise range of the Reynolds numbers.

KEY WORDS: fluid-structure interaction, flow instability, wave propagation, blood vessels.

СТАБИЛИЗАЦИЯ ТЕЧЕНИЙ ЖИДКОСТИ ПО МНОГОСЛОЙНЫМ ВЯЗКОУПРУГИМ ТРУБКАМ ПРИ УСЛОВИИ ЗАКРЕПЛЕНИЯ ИХ ВНЕШНЕЙ ПОВЕРХНОСТИ

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Неустойчивость течений жидкости в податливих трубках изучается в применении к течению крови и распространению пульсових волн в кровеносных сосудах как многослойных вязкоупругих трубках. Рассматривается случай осесимметричных возмущений и условий закрепления внешней поверхности трубки, что соответствует глубоким артериям, которые прикреплены к окружающим тканям. Показано, что в широком диапазоне чисел Рейнольдса неустойчивая жидкостная мода может быть стабилизирована путем выбора определенных значений вязкости и модулей упругости слоев стенки.

КЛЮЧЕВЫЕ СЛОВА: взаимодействие жидкость-стенка, неустойчивость течения, распространение волн, кровеносные сосуды.

СТАБІЛІЗАЦІЯ ТЕЧІЙ РІДИНИ КРІЗЬ БАГАТОШАРОВІ В'ЯЗКОПРУЖНІ ТРУБКИ ЗА УМОВОЮ ЗАКРІПЛЕННЯ ЇХ ЗОВНІШНЬОЇ ПОВЕРХНІ

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Нестійкість течій рідини в податливих трубках вивчається в застосуванні до течії крові і поширенню пульсових хвиль в кровоносних судинах як багатошарових в'язкопружних трубках. Розглядається випадок вісесиметричних збурень і умов закріплення зовнішньої поверхні трубки, що відповідає глибоким артеріям, які прикріплені до навколишніх тканин. Показано, що в широкому діапазоні чисел Рейнольдса нестійка рідинна мода може бути стабілізована шляхом вибору певних значень в'язкості та модулів пружності шарів стінки.

КЛЮЧОВІ СЛОВА: взаємодія рідина-стінка, нестійкість руху, розповсюдження хвиль, кровоносні судини.

1. Introduction. Fluid-structure interaction (FSI) is an important factor in the biofluid flows like blood motion in the arteries, capillaries and veins as well as in the distensible tubes of biomedical and technical systems [1–4]. When any disturbance of the flow and the produced variations in the hydrodynamic pressure and wall shear stress influence the stress distribution and wall movement and vice versa, there is a strong FSI between the fluid and solid. FSI and instability of the fluid flows in the compliant ducts have been thoroughly studied in application to the technical fluid-conveying systems, blood and urine flows through the vessels, air flow in the airways and other biosystems [5]. Instability of the steady flow of a viscous incompressible liquid near the compliant wall may produce wall oscillations, flow and pressure limitation phenomena, noise generation, fatigue of the wall material, and even destruction of the device. The wall oscillations are supported by the energy transfer from the fluid to solid at the interfaces [6]. Delay in the onset of the instability and laminar to turbulence transition, and stabilization of the unstable modes pose a challenging problem to both theoretical mechanics and mechanical engineering.

Flow stability in the isotropic compliant tubes has been in-depth studied in experiments and numerical computations on 1D and 2D mathematical models [6–11]. Though many physical and physiological phenomena of the flow instability in collapsed and noncollapsed tubes are explained, the problem is still of great interest in relation to different technical and biomedical applications, especially from the standpoint of sound generation and flow control in biological organisms and technical devices. In some cases the flow instability favours the transport efficiency, mixing of biological suspensions and polymers, and sound generation for communication between the organisms.

Flow-induced vibrations of arteries and veins can be detected by acoustic sensors on the human body over the superficial blood vessels and the problem in recognition of the pathological and the so-called innocent noise is and important problem of medical diagnostics [12]. Different problems of the flow instability in the blood vessels as multilayer tubes have been studied on the mathematical model of the stationary incompressible axisymmetric flow in a circular three-layer visocelastic tube at no displacement [13] and no stress [14] boundary conditions at the outer surface of the tube. It was found the unsteady fluid-based modes can be stabilized by the viscosity of the middle layer [13] and rigidity of the inner or outer layer [14]. In the both cases the sandwich-type material for the wall stabilizes the system. In this paper the possibility of the flow stabilization at the no displacement boundary conditions at wide variation of the Reynolds number within the physiological limits of the material parameters is studied.

2. Problem formulation

Stability of the Poiseuille flow of a viscous incompressible fluid in a multi-layer viscoelastic tube is considered. The wall is composed of three non-isotropic layers with thicknesses h_1, h_2, h_3, where $h_1 + h_2 + h_3 = h$ is the wall thickness, R and L are the inner radius and length of the tube (Fig.1). The outer surface of the tube r=R+h is tethered to the surrounding media which is supposed to be rigid.

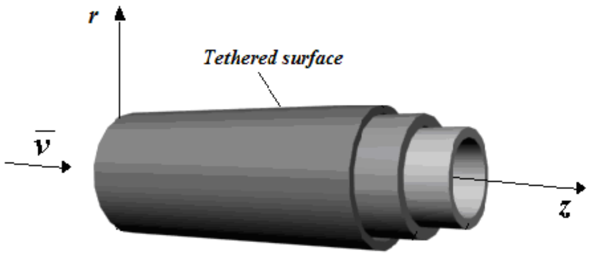


Fig1. Coordinate system and 3d model of the tube.

The incompressible Navier-Stokes equations governing the fluid flow are

$$\nabla \cdot \vec{v} = 0$$
$$\rho_f \frac{d\vec{v}}{dt} = -\nabla p + \mu_f \Delta \vec{v}, \quad (1)$$

and the mass and momentum conservation equations for the incompressible wall are

$$\nabla \cdot \vec{u}^{(j)} = 0,$$
$$\rho_w^{(j)} \frac{\partial^2 \vec{u}^{(j)}}{\partial t^2} = -\nabla p^{(j)} + \nabla \cdot \hat{\sigma}^{(j)}, \quad (2)$$

where ρ_f and $\rho_w^{(j)}$ are the mass densities of the fluid and solid layers, j = 1,2,3 correspond to the inner, medium and outer layers accordingly, $\vec{v}$ and μ_f are the fluid velocity and viscosity, $\vec{u}^{(j)}$ is the wall displacement, p and $p^{(j)}$ are the hydrostatic pressures in the fluid and solid layers, $\hat{\sigma}$ and $\hat{\sigma}^{(j)}$ are the corresponding stress tensors.

The viscoelastic body with parallel connection of the elastic and viscous properties (Voight model) has been considered for the wall layers:

$$\sigma_i^{(j)} = A_{ik}^{(j)} \varepsilon_k^{(j)} + \mu_w^{(j)} \frac{\partial}{\partial t} \varepsilon_k^{(j)} \quad (3)$$

where $\mu_w^{(j)}$ and $A_{ik}^{(j)}$ are the viscosity and matrix of elasticity coefficients, $\varepsilon_{ik}^{(j)} = \frac{1}{2}\left(\nabla_i u_k^{(j)} + \nabla_k u_i^{(j)}\right)$ is the strain tensor, $\vec{\sigma}^{(j)} = \left\{\sigma_{11}^{(j)}, \sigma_{22}^{(j)}, \sigma_{33}^{(j)}, \sigma_{23}^{(j)}, \sigma_{13}^{(j)}, \sigma_{12}^{(j)}\right\}$ is the stress vector, and $\vec{\varepsilon}^{(j)}$ is the similar strain vector.

The boundary conditions are the velocity and stress continuity conditions at the fluid-solid and solid-solid interfaces:

$$r = R \ : \ \vec{v} = \frac{d\vec{u}^{(1)}}{dt}, \ \vec{\sigma}_n = \vec{\sigma}_n^1, \ \vec{\sigma}_\tau = \vec{\sigma}_\tau^1 \quad (4)$$

$$r = R + h_1 \ : \ \vec{u}^{(1)} = \vec{u}^{(2)}, \ \vec{\sigma}_n^1 = \vec{\sigma}_n^2, \ \vec{\sigma}_\tau^1 = \vec{\sigma}_\tau^2 \quad (5)$$

$$r = R + h_1 + h_2 \ : \ \vec{u}^{(2)} = \vec{u}^{(3)}, \ \vec{\sigma}_n^2 = \vec{\sigma}_n^3, \ \vec{\sigma}_\tau^2 = \vec{\sigma}_\tau^3 \quad (6)$$

and the no displacement boundary condition at the outer surface of the tube

$$r = R + h \ : \ \vec{u}^{(3)} = 0, \quad (7)$$

where n and τ denotes the normal and tangential components of the stress tensor.

The mathematical problems (1)–(2) and (3)–(4) are coupled via the boundary conditions (4)–(7) and solution of the FSI problems (1)–(7) can be found as a superposition of the steady flow and small axisymmetric disturbance in the form of the normal mode [10–14]:

$$\{\vec{v}, p\} = \left\{\vec{v}^*, p^*\right\} + \left\{\vec{v}^\circ, p^\circ\right\} \cdot e^{st+ikz}$$
$$\left\{\vec{u}^{(j)}, p^{(j)}\right\} = \left\{\vec{u}^{(j)*}, p^{(j)*}\right\} + \left\{\vec{u}^{(j)\circ}, p^{(j)\circ}\right\} \cdot e^{st+ikz} \quad (8)$$

where $\vec{v}^\circ$, $\vec{u}^{(j)\circ}$, p°, $p^{(j)\circ}$ are the amplitudes of the corresponding disturbances, $k = k_r + ik_i$, $s = s_r + is_i$, s_i is the wave frequency, k_r is the wave number, s_r and k_i are temporal and spatial amplification rates. The steady part $\left\{\vec{v}^*, p^*\right\}$ of (8) is identified with Poiseuille flow.

The stable and unstable modes as well as the dependencies of the group velocities, spatial and temporal amplification rates on the model parameters have been computed using the numerical procedure developed in [10–14].

3. Computational results and discussions. The system parameters $\vec{v}^\circ$, $\vec{u}^{(j)\circ}$, p°, $p^{(j)\circ}$ are determined by the material constants, the non-dimensional parameter $\Gamma = v_x^* \sqrt{\rho_f / E^*}$ which is the ratio of the fluid to wall inertia forces [13], and flow regime described by the Reynolds number Re= $\rho_f v_x^* R / \mu_f$. The parameter Γ can be re-written in the form $\Gamma = v_x^* / (c_w \sqrt{\rho})$ where c_w is the wave velocity in the elastic wall, $\rho = \rho_w / \rho_f$ is the relative density of the wall.

The blood vessel wall exhibits viscoelastic transversal isotropic properties with different Young modules and Poisson ratios in the longitudinal and tangential directions. It is important that the tree layers may have different material parameters either in the normal state or at the pathology (see the detailed review in [13]). The inner layer is the thinner one but its thickness can be significantly increased by local hemodynamic factors and arterial blood pressure that influence the endothelial function and penetration of potentially atherogenic particles, developing intimal fibrosis and atheroma. Therefore, the relative thickness h_1/h can vary between 2–3% and 30–40%. Medium layer thickening is observed at atherosclerosis, hypertension, and wall remodeling caused by local hydrodynamic and global biochemical factors from 25–30% to 40–50%. The remodeling depends on type and location of the wall and can be directed outward and inward resulting in the narrowing or enlarging the external diameter of the artery [16]. Hypertrophic/hypotrophic remodeling is characterized by increased/decreased wall-to-lumen ratio. The intermediate eutrophic remodeling is characterized by changes in the vessel diameter at constant wall-to-lumen ratio. In that way the relative thicknesses h_j/h may vary in quite wide physiological ranges that have been taken into account at the computations.

Characteristic elastic modulus for the wall layers is E*~10^5–10^6 Pa for the normal elastic and muscle type arteries while in the cases of the atherosclerotic plaque and calcium accumulation in the wall the local rigidity may be higher E*~10^7–10^8 Pa [13]. Both isotropic and transversely isotropic materials for the wall layers have been studied. In that way the matrix A_{ik} has been introduced in the form

$$\left(A_{ik}\right)^{-1} = \begin{pmatrix} (E_2)^{-1} & -\nu_2(E_2)^{-1} & -\nu_2(E_2)^{-1} & 0 & 0 & 0 \\ -\nu_2(E_2)^{-1} & (E_1)^{-1} & -\nu_1(E_1)^{-1} & 0 & 0 & 0 \\ -\nu_2(E_2)^{-1} & -\nu_1(E_1)^{-1} & (E_1)^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & (G_2)^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & (G_1)^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & (G_1)^{-1} \end{pmatrix}$$

When E_1=E_2, G_1=G_2, $\nu_1 = \nu_2$ for each layer j=1,2,3, the wall is isotropic, while when those values are equal in different directions but different for the layers, the wall in anusotropic in the radial direction. In this study two types of anisotropy have been considered: {E_1=2E*, E_2=20E*} and {E_1=20E*, E_2=2E*} that corresponds to more pronounced circumferential and longitudinal rigidity correspondingly.

According to numerical computations at different boundary conditions [13–15], the Poisson ratio does not influence the unstable modes and group velocity and can be accepted as $\nu^{(j)}$~0.4–0.5 depending on the size and wall thickness of the artery. The medium and small arteries are practically incompressible while the larger ones possess some compressibility due to developed system of the small blood vessels *vasa vasorum* supplying the muscle cells in the medium layer.

Other material parameters have been chosen according to physiological data in normalcy and pathology within the following ranges $\rho_f = 1050-1100\ \mathrm{kg/m^3}$, $\rho_w^{(j)} = (0.9-1.3)\rho_f$, $\mu_f =$ (1.5–20) $\cdot 10^{-3}$ Pa·s, μ_w =0.01–1 Pa·s, h/R=0.15–1.5.

By varying the set of material properties of the wall layers, one can either increase or reduce the conductivity of the system in relation to the stationary and wave flow. As it was shown by the numerical calculations [13–15], the viscosity and shear modules of the wall layers have the most significant impact on the absolute and convective instability of the system. Here the computational results of the influence of the wall viscosity on the temporal amplification rate s_r at different Re numbers are presented in Fig.2 at $\Gamma = 1$. The non-dimensional wall viscosity $\mu = \mu_w / \mu_f$ is varied from zero (purely elastic wall) to the values when asymptotic behavior of the dependence $s_r(\mu)$ becomes clear. In Fig.2 the dependencies $s_r(\mu)$ are plotted at $\mu \in [0,3]$ highlighting the transition from the elastic to viscoelastic wall and showing the influence of the wall viscosity on the system stability. When s_r is positive the system exhibits temporal instability with exponentially increased amplitudes of the disturbances. The Re number varied as Re=0–500 but the curves $s_r(\mu)$ are presented in Fig.2 at Re=10–40 only to demonstrate the tendency of the system stability at relatively low Re numbers.

As it is shown in Fig.2, the fluid-filled elastic tube possess one unstable mode with $s_r = 0.56...$ and this mode can be stabilized by increase of the viscosity of the second layer (Fig.2c) or all three layers simultaneously (Fig.2a). It is clear that stabilization is provided by viscosity of the middle layer because the inner (Fig.2b) and outer (Fig.2d) layers do not stabilize to system at Re<50. The stabilization can be achieved at any Re number by any viscosity of the inner layer $\mu > 0.1$ (Fig.2c), while when all the layers become viscous the stabilizing effect of the middle layer prevail over the destabilizing effects of the outer and inner layers within short range of the relative viscosity values $\mu \in]0.1, 2[$ depending on the Re numbers (Fig.2a).

The same dependencies for the higher Re numbers Re=50–200 are presented in Fig.3. Note the studied values of the Reynolds numbers provide stable steady flow in the rigid tube (Poiseuille flow) and the instability is connected with improper combination of

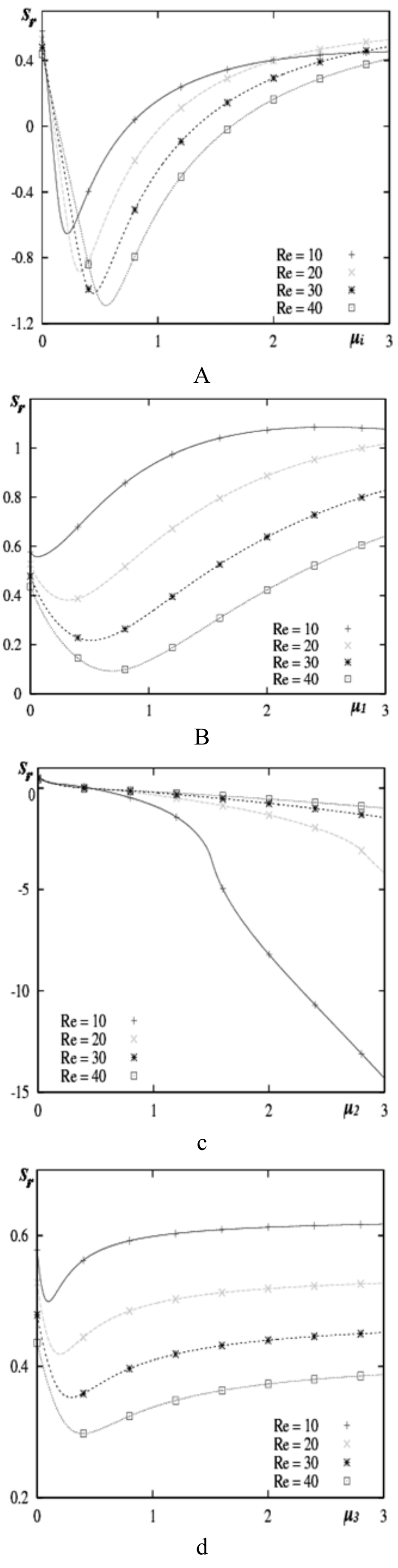


Fig.2. Temporal amplification rate of the most unstable mode versus the viscosity of the three (a), inner (b), medium (c) and outer (d) layers at Re = 10, 20, 30, 40.

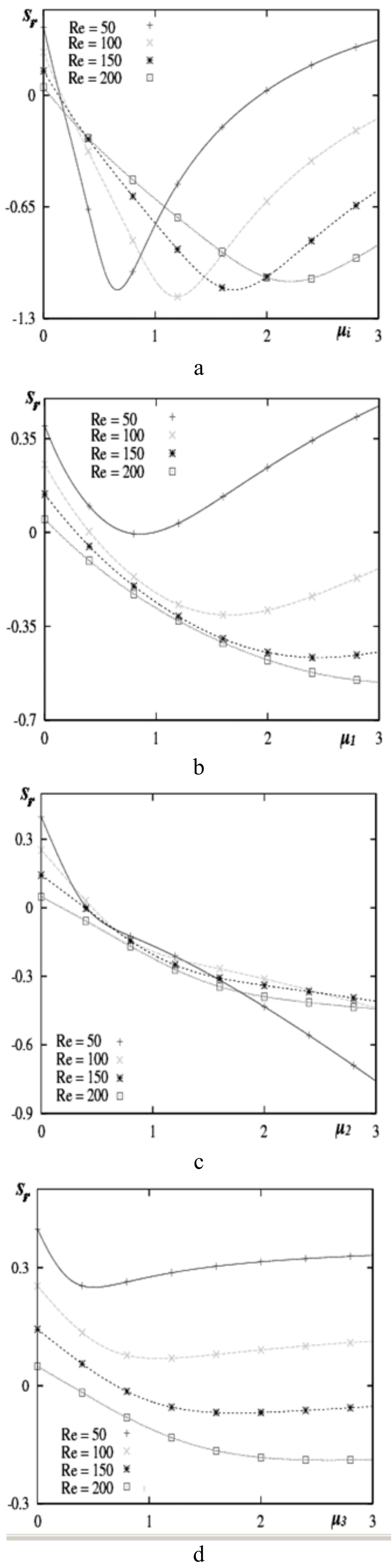
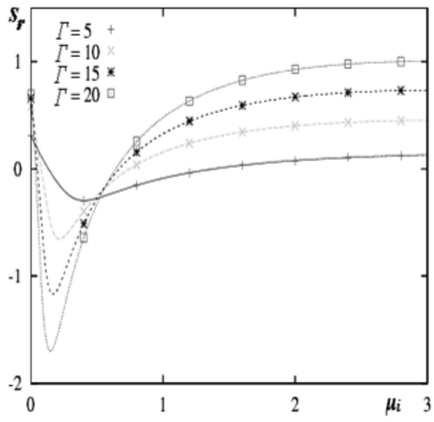
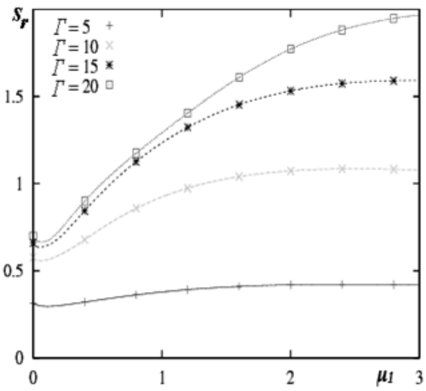


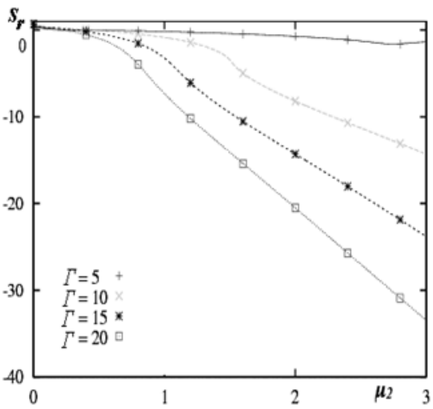
Fig.3. Temporal amplification rate of the most unstable mode versus the viscosity of the three (a), inner (b), medium (c) and outer (d) layers at Re = 50, 100, 150, 200.



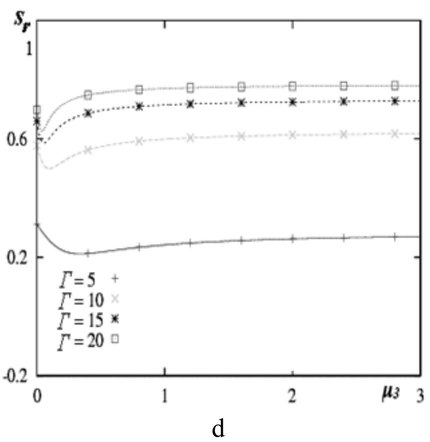
a



b



c



d

Fig. 4. Temporal amplification rate of the most unstable mode versus the viscosity of the three (a), inner (b), medium (c) and outer (d) layers at G=5,10,15,20 and Re=10.

the fluid and solid properties and FSI that may promote rapid spatial and temporal growth of any small disturbance.

It is also shown the dependence $s_r(\mu_2)$ is monotonously decreasing and provides fast stabilization of the system while the dependencies $s_r(\mu_1)$ and $s_r(\mu_2)$ becomes negative with increasing the corresponding viscosity within certain region of the viscosity, though the region becomes larger when Re increases and Re ≥ 60 for the inner layer (Fig.3b), Re ≥ 120 for the outer layer (Fig.3d). In the case of the isotropic wall the combination of the stabilizing effect of the viscosity of the middle layer and destabilizing influence of the viscosity of the outer and inner layer leads to stabilization at wide ranges of the relative viscosity depending on the Re number (Fig.3a).

In that way it was shown that the sandwich-type materials composed by the elastic layers with a viscoelastic layer in between [13–15], are excellent candidates for the flow stabilizing coatings in the compliant tubes at a wide range of the Reynolds numbers.

The influence of the non-dimensional parameter *Γ* in the flow stabilization by the viscosity of different layers is presented in Fig. 4 for Re=10. The stabilization influence of the viscosity of the middle layers still exists and even becomes more pronounced while *Γ* increases, though the destabilizing effect of the inner and outer layers also increases for the higher values of *Γ*. As a result, the net stabilizing effect of the viscosity of the three layers appear within the more and more narrow ranges of μ while *Γ* increases. The same regularities have been found at the higher Reynolds numbers Re=10–500. Increased Re number enhances the successful range of the wall viscosities stabilizing the system, while increased *Γ* acts in the opposite direction, so at any set of the non-dimension parameters {Re, *Γ* } governing the flow the physically reasonable values of the wall viscosities $\mu_{1,2,3}$ can be found for the flow stabilization purposes.

4. Conclusions.

It was shown that the system instability strongly depends on the rheological properties of the fluid and wall. The rigidities and viscosities of the wall layers produce the most prominent effects on the temporal amplification rates of the unstable mode. At different flow regimes the unstable fluid-based mode can be stabilized by an increase in the viscosity of the inner layer of all three layers of the wall. In means, the mode can be damped by the viscous wall or the sandwich-type coating composed by a viscous layer located between two elastic layers with low viscosities. The effect is kept within the range of low and intermediate Re=10–500. It is proved at any flow regimes and fluid rheology a successful set of material parameters of the layers stabilizing the system can be found for either isotropic or transversely isotropic wall. The obtained results can be used for detailed understanding the wall remodeling of the blood vessels directed to the flow stabilization at some stages of development of the vascular pathology, as well as for the flow stabilization in distensible tubes of different industrial and biomedical devices.

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