UDC 681.5.015.4

ON STRUCTURAL─PARAMETRIC IDENTIFICATION OF TIME─DELAY SYSTEMS FROM REAL IMPULSE RESPONSE

Glushko O.V., Tkachev R.U.

Donbass State Technical University, Alchevsk, Ukraine

In the paper, the new approach to structural-parametric (SP) identification of time-delay systems from real impulse response is proposed. The approach is based on representation of experimental data that corresponds to the free motion stage of a system in a form of formal Laurent series with the subsequent approximation using mathematical apparatus of continuous fractions, which provides the determination of the model's structure. The determination of parameters is realized then with solution of the system of transcendental equation formed using transient process formula on the basis of other stages' experimental data and defined model's structure.

KEY WORDS: time-delay systems, impulse response, identification, Laurent series.

О СТРУКТУРНО-ПАРАМЕТРИЧЕСКОЙ ИДЕНТИФИКАЦИИ СИСТЕМ С ЗАПАЗДЫВАНИЕМ ПО ДЕЙСТВИТЕЛЬНОМУ ИМПУЛЬСНОМУ ОТВЕТУ

Глушко О.В., Ткачев Р.У.

В работе предложен новый подход к структурно-параметрической (СП) идентификации систем с запаздыванием по действительному импульсному ответу. Предложенный подход основан на представлении экспериментальных данных, соответствующих свободному движению системы в форме формальных рядов Лорана с последующей аппроксимацией с использованием непрерывных дробей, что позволяет определить структуру модели. Затем проводится определение параметров модели из системы трансцендентных уравнений, полученных путем использования формулы переходных процессов на основе экспериментальных данных о других состояниях системы и по известной структуре модели.

КЛЮЧЕВЫЕ СЛОВА: системы с запаздыванием, импульсный ответ, идентификация, ряды Лорана.

ПРО СТРУКТУРНО-ПАРАМЕТРИЧУ ІДЕНТИФІКАЦІЮ СИСТЕМ З ЗАПІЗНЕННЯМ ПО ДІЙСНІЙ ІМПУЛЬСНІЙ ВІДПОВІДІ

Глушко О.В., Ткачов Р.У.

В роботі запропоновано новий підхід до структурно-параметричної (СП) ідентифікації систем з запізненням по дійсній імпульсній відповіді. Запропонований підхід заснований на уявленні експериментальних даних, які відповідні вільному руху системи, у формі формальних рядів Лорана з подальшою апроксимацією з використанням неперервних дробів, що дозволяє визначити структуру моделі. Потім проводиться визначення параметрів моделі з системи трансцендентних рівнянь, які отримані шляхом використання формули перехідних процесів на основі експериментальних даних про інші стани системи і за відомою структурі моделі.

КЛЮЧОВІ СЛОВА: системи з запізненням, імпульсна відповідь, ідентифікація, ряди Лорана.

1. Introduction. At present, most of the works on system identification are still based on parametric models so that identification in these works is a process of determining of unknown parameters of a model with predefined structure [1, 2].

Sometimes parametrical identification is not acceptable as, generally, the more model's structure differs from that of the "true system" the less conformity between model response and experimental data can be achieved and in relevant cases even slight structural divergences of the model may lead to the significant distortion of model parameters so that structural identification is often needed [1].

Although many works in the field of structural and structural-parametric (SP) identification has been published recently, there still not exists a well-developed universally recognized SP-identification method neither for nonlinear dynamical systems nor for linear ones. Such a way a further research in the field of SP-identification is vital for control theory [3, 4].

The most SP-identification methods are based on using frequency-domain data or step-response time-domain data that in defined situations makes it difficult(or even impossible) to apply them for identification of real-life systems due to the physical restrictions. Quite often using of impulse responses may solve the problem so that because of reasons listed above it can be stated that the present work is devoted to the actual problem of system identification.

In this paper, we propose the new approach to SP-identification of time-delay systems from real impulse response. The approach is based on representation of experimental data that corresponds to the free motion stage of a system in a form of formal Laurent series with the subsequent approximation using mathematical apparatus of continuous fractions, which provides the determination of the model’s structure. The determination of parameters is realized then with solution of the system of transcendental equation formed using inverse Laplace transform formula on the basis of other stages’ experimental data and defined model’s structure.

2. Main results. In real-life systems, it is usually impossible to generate a perfect impulse and therefore real impulses such as rectangular, trapezoidal, saw tooth, triangular at alias are used. If the real pulse is short enough compared to the impulse response it can be treated as an ideal one because in that situation the resulting system response is close to the ideal impulse response. However, input signal should be of the constant energy, so that the shorter signal is the higher power it should have that makes using very short impulses impossible due to the physical restrictions.

The problem of identification based on the real impulse responses is quite a complicated one as any change of character of an input signal being not taken into account leads to the structural error that in it's turn leads to the distortion of both zeros and poles of identified models.

The character of the impulse response on the example of trapezoidal input signal is shown in Fig. 1. Stage I of the impulse response in Fig. 1 is the reaction on a ramp-up signal, II – is the reaction on a constant signal, III – is the reaction on a ramp-down signal. All stages of a process except first one are influenced by the initial phase state of a system caused by the previous stage.

Identification based on real impulse signals can be realized in two stages: 1) determination of model's poles using the down sampled backward sequence of output value in the range that corresponds to undisturbed motion of the system and 2) determination of unknown zeroes from distributed motion data using known poles.

As information on the technological process’s parameters in modern control systems is usually introduced in a digital form, it is convenient to use the indirect approach which implies two consequent stages: 1) determination of the discrete-time model, 2) retrieving of the desired continuous model on the base of discrete-time one.

Within the framework of mathematical apparatus of the discrete-time systems the problem of structural-parametric identification can be formulated as the problem of effective approximation of the power series

$$f(z) = c_0 + c_1 z^{-1} + c_2 z^{-2} + c_3 z^{-3} \ldots, \qquad (1)$$

where $z = e^{Ts}$, $s = \sigma + j\omega$.

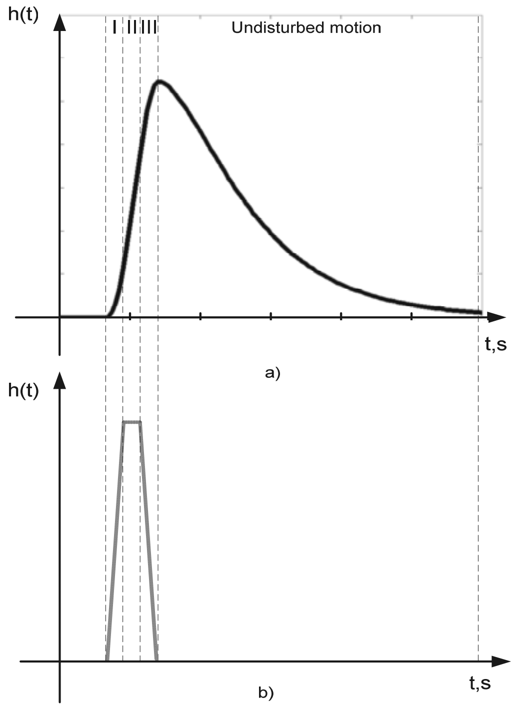

Fig.1. Real impulse response (a) caused by the trapezoidal impulse signal (b).

Series (1) are formal Laurent series that makes it possible to use the analytic theory of continued fractions for the approximation (it should be noted that the idea of using mathematical apparatus of continued fractions for identification of dynamical systems was proposed by V. Ya. Kartashov in [5]). Selection of an order of approximation is the problem to solve at this stage of identification.

The Rutishauser method was chosen to be the method of representing analytic functions by continued fractions. It is determined by the formula [6]:

$$f(z) \approx \frac{c_0}{1-}\,\frac{q_1^{(0)}z^{-1}}{1\;-}\,\frac{e_1^{(0)}z^{-1}}{1\;-}\,\frac{q_2^{(0)}z^{-1}}{1\;-}\,\frac{e_2^{(0)}z^{-1}}{1\;-}\ldots =$$

$$= \cfrac{c_0}{1-\cfrac{q_1^{(0)}z^{-1}}{1-\cfrac{e_1^{(0)}z^{-1}}{1-\cfrac{q_2^{(0)}z^{-1}}{\ddots\, 1-\cfrac{q_{n-1}^{(0)}z^{-1}}{1-e_{n-1}^{(0)}z^{-1}}}}}} = \frac{P(c_0,\{e^{(0)}\},\{q^{(0)}\})}{Q(\{e^{(0)}\},\{q^{(0)}\})} \qquad (2)$$

where $e_m^{(n)} \in \mathbb{C}$, $q_m^{(n)} \in \mathbb{C}$, $f(z) \in \hat{\mathbb{C}}$, $\hat{\mathbb{C}} = \mathbb{C} \cup [\infty]$; $\{e_m^{(n)}\}$ and $\{q_m^{(n)}\}$ are sequences determined with following relations: $e_m^{(n)} = q_m^{(n+1)} - q_m^{(n)} + q_{m-1}^{(n+1)}$; $q_{m+1}^{(n)} = \frac{e_m^{(n+1)}}{e_m^{(n)}} q_m^{(n+1)}$;

m=1,2,3…; n=0,1,2,3…; except $e_0^{(n)} = 0$ and $q_1^{(n)} = \frac{c_{n+1}}{c_n}$.

As it is implied that in (2) $e_m^{(n)} \neq 0$ and $q_m^{(n)} \neq 0$ (for m=1,2,3…), the sequence $\{c_n\}$ should be shifted to the first nonzero element and the resulting continued fraction should be multiplied by z^{-d} according to the delay theorem, where d is a shift of the lattice function.

The determination of continued fraction coefficients can be realized by calculation of an identification matrix

$$\begin{matrix} 0 & a_{1,1} & a_{1,2} & \cdots & a_{1,n-2} & a_{1,n-1} \\ 0 & a_{2,1} & a_{2,2} & \cdots & a_{2,n-2} & \\ \cdots & \cdots & \cdots & \cdots & & \\ 0 & a_{n-1,1} & & & & \\ 0 & & & & & \end{matrix}, \quad (3)$$

where n is a length of $\{c_n\}$, $a_{i,1} = \frac{c_{i+1}}{c_i}$, and if j≥2:

$a_{i,j} = \frac{a_{i+1,j-1}}{a_{i,j-1}} a_{i+1,j-2}$ for even j values, and

$a_{i,j} = a_{i+1,j-1} - a_{i,j-1} + a_{i+1,j-2}$ for odd j values.

The calculation of matrix (3) is finished when coefficient of the first row equal to zero is achieved (in practice the coefficient absolute value should be several orders less then the previous one in the same row) that corresponds to small absolute values of the residual series. Such a way the order of a model is defined.

Then, basing on the fist row elements up to the zero-element a continued fraction is formed. The discrete Laplace image of the process is then:

$$G(z) = \left(\frac{c_0}{1} \frac{a_{1,1} z^{-1}}{1 -} \cdots \frac{a_{1,n} z^{-1}}{- \quad 1} \right) z^{-d} = \frac{P_n}{Q_n} z^{-d},$$

where n – order of the Laplace image; d – discrete time delay.

To provide the exact structure equivalence of retrieved and original CTF the matched Z-transform is used which is defined by formula [7]:

$$s_i = \frac{\ln(z_i)}{T_s}, \quad (4)$$

where s_i are roots of CTF; z_i are roots of DTF; T_s is sampling period, s.

The shift of the lattice function which causes distortion of zeroes of the transfer function doesn't affect it's poles so that using formula (4) we'll retrieve exact(undistorted) poles of the model that need no correction.

It should be noted that the transformation should be carried out within the general frequency band, limited by Nyquist frequency, so that negative roots of z-plane are looped off during the mapping to the s-plane.

Determination of the most appropriate values of zeros using poles determined before can be realized with solution of the system of equations formed on the basis of transient process formula.

$$\begin{cases} \sum_{i=0}^{n_1} c_{1,i} e^{-k_i t_1} + \sum_{i=0}^{n_1} r_{1,i} e^{-k_i t_1} = y(t_1) \\ \sum_{i=0}^{n_2} c_{2,i} e^{-k_i t_2} + \sum_{i=0}^{n_2} r_{2,i} e^{-k_i t_2} = y(t_2) \\ \quad \cdots \\ \sum_{i=0}^{n_m} c_{m,i} e^{-k_i t_m} + \sum_{i=0}^{n_m} r_{m,i} e^{-k_i t_m} = y(t_m) \end{cases}, \quad (5)$$

where n_j is the order of Laplace transform image of a process at time t_j; $c_{j,i}$ are unknown numerators of simple fractions that in sum representing image of a process at time t_j; t_j is time of a sample, s; $k_{j,i}$ are known poles of a process at time t_j; $r_{j,i}$ are known coefficients, reflecting the initial state at the beginning of a process at time t_j; $y(t_j)$ is the value of the output coordinate at time t_j.

It should be noted that the time of a sample is supposed to be the time in relation to the beginning of the stage.

Systems of equations like (5) can be solved with numerical methods.

3. Example of identification. To demonstrate the proposed method consider the identification problem of object with transfer function of true system

$$W_{t.s.}(s) = \frac{0,5s+1}{(s+1)(2s+1)} e^{-1,33s}. \quad (6)$$

It is assumed that ADC sampling period $T_s = 0,1$s, and the input signal is a real trapezoidal impulse defined by the formula:

$$\begin{aligned} y_{in}(t) &= 2t(1-\sigma(t-0,5)) + \sigma(t-0,5) - \\ &- \sigma(t-1) - 2t(\sigma(t-1) - \sigma(t-1,5)) \end{aligned},$$

where σ (t) is Heaviside step function.

The backward down sampled (to T_s =1s) sequence of the system response values is {y} = {0,0053; 0,0087; 0,0143; 0,0234; 0,0380; 0,0611; 0,0966; 0,1481; 0,2137; 0,2697 …}

Discrete Laplace image of the process is then

$$F(z)=\frac{0,0380}{1-\dfrac{1,6088z}{1+\dfrac{0,0275z}{1-2,7856z}}}=\frac{-0,02339z+0,00848}{(z-0,6065)(z-0,3679)}$$

Using (4) we define poles of model's continuous transfer function:

$$s_1^p=\ln(0,6065)\approx -0,5\,;\; s_2^p=\ln(0,3679)\approx -1\,.$$

Then the continuous transfer function of the model can be represented as

$$F(s)=\frac{N_1s^2+N_2s+N_3}{(s+1)(s+0,5)}e^{\tau},$$

where $N_1=r_1+r_2+r_3$; $N_2=1,5r_1+0,5r_2+r_3$; $N_3=0,5r_1$; r_1, r_2, r_3 are unknown coefficients that can be determined by solution of the following system of transcendental equations:

$$\begin{cases} 2r_2+4r_3-2r_2e^{d_\tau T_s-t_1}e^{\tau_e}-4r_3e^{0,5(d_\tau T_s-t_1)}e^{0,5\tau_e}+ \\ \qquad +2r_1(t_1-d_\tau T_s-\tau_e)=y_1 \\ r_1+(r_2-l_1+v_1)e^{-t_2}e^{\tau_e}+(r_3+l_2-v_2)e^{-0,5t_2}e^{0,5\tau_e}=y_2 \\ r_1+(r_2-l_1+v_1)e^{-t_3}e^{\tau_e}+(r_3+l_2-v_2)e^{-0,5t_3}e^{0,5\tau_e}=y_3 \\ r_1+(r_2-l_1+v_1)e^{-t_4}e^{\tau_e}+(r_3+l_2-v_2)e^{-0,5t_4}e^{0,5\tau_e}=y_4 \\ l_1=5r_1+3,2131r_2+4r_3 \\ l_2=6r_1+4r_2+4,8848r_3 \\ v_1=2r_1+r_2+2r_3 \\ v_2=6r_1+4r_2+5r_3 \end{cases}$$

where $\tau=d_\tau T_s+\tau_e$ is unknown time-delay; τ_e is time-delay sampling error ($0\le\tau_e<T_s$); d_τ is shift of the lattice function to the first nonzero element (d_τ =13).

Let's select t_1 =1,8s (from the beginning of the process); t_2 =0,2s; t_3 =0,4s ; t_4 =0,5s; (t_2, t_3, t_4 - time from the beginning of the second stage). Corresponding values of output coordinate: y_1 =0,0584; y_2 =0,1129; y_3 = 0.1687; y_4 =0,1967.

The solution of the system: r1 ≈ 1.0; r2 ≈ 0.5; r3 ≈ -1.5; $\tau_e\approx 0{,}03$s. The model is then:

$$W_{m.}(s)=\frac{0,25s+0,5}{(s+1)(s+0,5)}e^{-1,33s}\,.$$

Thus, transfer function (6) was retrieved with a high accuracy.

4. Conclusions. The new approach to SP-identification of time-delay systems from real impulse response was proposed. The efficiency of the method is demonstrated on the example of identification of the system with the true system (6).

Advantages of the proposed approach include high accuracy of identification and possibility to realize complex calculations using modern controllers although this is not a trivial task. Main disadvantage is using the a priori knowledge of the input impulse signal that is the fact that divergences between real input signal and programmed one are not taken into consideration.

REFERENCES

1. Ljung L. *System identification* – theory for the user. 2nd ed. PTR Prentice Hall, Upper Saddle River, – New Jersey. -1999. – 609 p.
2. Ljung L., Söderström T. *Theory and practice of recursive identification.* Cambridge, Massachusetts, London, MIT Press, 1987. – 541 p.
3. Karabutov N.N. *Structural identification: Analysis of dynamical structures*. Moscow, Moscow State Industrial Univ. Publ. – 2008. – 160 p. (in Russian).
4. Nikulchev E. V. *Identification of dynamic systems based on symmetry of reconstructed attractors*. – Moscow, Moscow State Univ. of Publ. Arts, 2010. – 100 p. (in Russian).
5. Kartashov V. Ya. Is the equivalence of discrete models possible? *Promichlennie sistemi i kontrolleri*. – 2006. – N 8. – P.40–44. (in Russian)
6. Baker G. A., Graves-Morris P. *Pade approximants*. London, Addison-Wesley Publishing, 1981. – 502 p.
7. Kuo B. *Digital control systems*. London, Oxford University Press, 1992. – 448 p.