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DURABILITY INDEXES OF ELEMENTS OF STRUCTURES ESTIMATING BASED ON THE THEORY OF CREEP INITIAL-BOUNDARY VALUE PROBLEMS

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Estimation of durability indexes taking into account dispersions of the operating conditions of elements of structures based on the theory of creep initial-boundary-value problems is presented. The estimation of durability indexes is reduced to the elements of structures that failure the time probably density determination using the data on the failure time depending on operating conditions and the probably densities functions of the operating conditions. Failure time depending on the operating conditions for the structure elements is determined by the numerical solution of the initial-boundary-value problem of the creep theory. The example of the proposed methods for the estimations of the durability indexes of the internal pressured pipes under high-temperature creep conditions is discussed. It is shown that the structure elements durability estimation based on the mean life value is inflated in the sense that it corresponds to the low probability of the failure absent during the operation time. The gamma-percentile life is a more adequate assessment of the structure elements durability which allows finding out the structure elements operating time corresponded to the given allowable probability of the failure absence.

KEY WORDS: mechanics of constructions, durability, creep theory, initial-boundary value problem.

ОЦЕНКА ИНДЕКСОВ ВЫНОСЛИВОСТИ ЭЛЕМЕНТОВ СТРУКТУР НА ОСНОВЕ НАЧАЛЬНО-КРАЕВЫХ ЗАДАЧ ТЕОРИИ ПОЛЗУЧЕСТИ

Ромашов Ю.В.

В работе проведено определение вероятностных показателей долговечности эксплуатирующихся в условиях ползучести элементов конструкций. В основе предлагаемого подхода к оценке показателей долговечности лежит решение дифференциальных уравнений с начальными и граничными условиями – начально-краевых задач теории ползучести для определения времени разрушения при заданных свойствах материала и эксплуатационных нагрузках без учета вероятного разброса их значений и последующая обработка результатов такого решения методами теории вероятностей. В результате применения предлагаемого подхода получен закон плотности распределения времени достижения предельного состояния элементов конструкций, эксплуатирующихся в условиях ползучести, который позволяет определять показатели долговечности, а именно гамма-процентный и средний ресурс.

КЛЮЧЕВЫЕ СЛОВА: механика конструкций, долговечность, теория ползучести, начально-краевая задача.

ОЦІНКА ІНДЕКСІВ ВИТРИВАЛОСТІ ЕЛЕМЕНТІВ СТРУКТУР НА ОСНОВІ ПОЧАКОВО-КРАЙОВИХ ЗАДАЧ ТЕОРІЇ ПОВЗУЧОСТІ

Ромашов Ю.В.

В роботі проведено визначення ймовірнісних показників довговічності елементів конструкцій, які експлуатуються в умовах повзучості. В основі запропонованого підходу до оцінки показників довговічності лежить вирішення диференціальних рівнянь з початковими і граничними умовами - початково-крайових задач теорії повзучості для визначення часу руйнування при заданих властивостях матеріалу та експлуатаційних навантаженнях без урахування ймовірного розкиду їх значень, а також подальша обробка результатів такого рішення методами теорії ймовірностей. В результаті застосування запропонованого підходу отримано закон щільності розподілу часу досягнення граничного стану елементів конструкцій, що експлуатуються в умовах повзучості, який дозволяє визначати показники довговічності, а саме гамма-відсотковий і середній ресурс.

КЛЮЧОВІ СЛОВА: механіка конструкцій, довговічність, теорія повзучості, початково-крайова задача.

1. Introduction.

Durability indexes required for the failure time estimation taking into account the probable dispersion are necessary for the risk assessment of a possible accident of some potential accidents with operating technical systems, especially in the nuclear energy [1] and chemistry [2] industries. Such durability indexes are the gamma-percentile life (the operating time that corresponds to the γ-percentile probability of the failure absence) and the mean life (the mathematical expectation of the failure time) [3].

Analyses of the solid bodies deforming at the high-temperature creep conditions led to partial differential equation systems with initial and boundary conditions, i.e. to the initial-boundary value problems [4]. Such approach is widely used and it allows estimation of the value of the failure time under the prescribed material properties and external forcing without their probable dispersion (e.g. see [5]). The present paper deals with estimation of the durability indexes basing on solution of the initial-boundary value problems of creep theory for elements of structures operated under the high-temperature creep accounting for probable dispersion of the operating conditions.

2. The general approach to the durability indexes estimation. Durability indexes presented as a gamma-percentile t_γ and a mean $\langle t_* \rangle$ life of structural elements can be defined using a probability density function $f = f(t_*)$ of a failure time t_* in the form [3]

$$t_\gamma : \int_0^{t_\gamma} f(t_*)\,dt_* = 1 - \frac{\gamma}{100}, \; \langle t_* \rangle = \int_0^{\infty} t_* f(t_*)\,dt_* . \quad (1)$$

Possible dispersion of the failure time is a consequence of the dispersion of the operating conditions of the structural elements, especially the material properties and dispersion of the external forces. Let us introduce the operating conditions (i.e. material properties and external forces) as elements $\mathbf{z}$ of the space Z. It is obvious that the failure time t_* depends on the operating conditions that can be represented as the mapping

$$t_* = t_*(\mathbf{z}). \quad (2)$$

Let Z is the finite N_Z-dimensional space with basis $\mathbf{Z}_1$, $\mathbf{Z}_2$, ..., $\mathbf{Z}_{N_Z}$. Then any element $\mathbf{z} \in Z$ can be presented as

$$\mathbf{z} = \sum_{i=1}^{N_Z} z_i \mathbf{Z}_i . \quad (3)$$

Here z_i are the coordinates in the $\mathbf{Z}_i$ basis. With regard to the expression (3), a mapping (2) will takes the form of the function of the many variables

$$t_* = t_*(z_1, z_2, \ldots, z_{N_Z}). \quad (4)$$

The dispersion of the operating conditions presented for the elements of structures by the element $\mathbf{z} \in Z$ can be represented as the consequence of the random nature of the z_i coordinates. So, with regard to the expression (4), the failure time t_* is the function of the random arguments. An evaluation of the failure time possible dispersion requires the data of possible dispersions of the material characteristics and the external forces, i.e. the probability density functions of the z_i coordinates. Let us denote $\zeta_i = \zeta_i(z_i)$, $i = 1, 2, \ldots, N_Z$ the probability density functions of the z_i coordinates (i.e. the possible dispersions of the material characteristics and the external forces) such that

$$\int_{z_i^{(\min)}}^{z_i^{(\max)}} \zeta_i(z_i)\,dz_i = 1 . \quad (5)$$

Here $z_i^{(\min)}$ and $z_i^{(\max)}$ are the minimal and maximal values of the z_i coordinate. The probability density function of the system of the uncorrelated random variables z_i with the regard to the expression (5) takes the form [6]

$$\zeta = \zeta(z_1, z_2, \ldots, z_{N_Z}), \qquad \zeta(z_1, z_2, \ldots, z_{N_Z}) = \prod_{i=1}^{N_Z} \zeta_i(z_i). \quad (6)$$

The probability density functions $f = f(t_*)$ of the values of the failure time function (4) of the random arguments z_i with the probability density function (6) can be determined according to the theory of probability [6] as following

$$f(t_*) = \frac{dF(t_*)}{dt_*}, \qquad F(t_*) = \underset{D(t_*)}{\iint \ldots \int \ldots} \zeta(z_1, z_2, \ldots, z_{N_Z})\,dz_1 dz_2 \ldots dz_{N_Z}. \quad (7)$$

Here $D(t_*)$ denotes the range of those values z_i for which the condition $t_*(z_1, z_2, \ldots, z_{N_Z}) < t_*$ is satisfied. In the simplest case when $N_Z = 1$ and $z_1 \equiv z$ instead of a function of many variables (4), we will have the failure time as a function of one variable

$$t_* = t_*(z). \quad (8)$$

The probability density function of the random variable z takes the more simple form than (6) that is also a function of one variable

$$\zeta = \zeta(z). \quad (9)$$

Probability density functions $f = f(t_*)$ of values of the failure time function (8) of the random argument z with probability density function (9) can be determined according to the theory of probability [6] as following

$$f(t_*) = \zeta(z(t_*)) \left| \frac{dz(t_*)}{dt_*} \right|, \quad (10)$$

where $z = z(t_*)$ is the inverse function of (8).

Estimation of the durability indexes can be reduced to the probability density function for failure time t_*, that is evident from the expressions (1). The probability density function for the failure time t_* requires representation of the failure time as a function of many variables (4) (or a function of one variable in the simplest case $N_Z = 1$) and the probability density functions of the z_i coordinates (or the one coordinate z) as can be seen from the expression (7) for general case (or expression (10) for the simplest case). The probability density functions of z_i coordinates are predetermined by the probably dispersions of the operating conditions o the structural elements including the material properties and the external forces. Such the data are commonly known form the operating experience of the corresponding technical systems. In that way, the dispersions of characteristics of the materials of the structural elements are covered by the finite intervals and their probability density functions can be defined via testing the specimens made from different materials. The external forces can be defined using the data on the fields measured in similar technical systems (nuclear reactors, steam boilers, chemical equipment and others) and adjusted by taking into account the expert assessments. The representation of the failure time for the structural elements as a function of many variables (4) or a function (8) of one variable can also be established by natural experiments at the operating plants, but the approach is associated with high costs and potential dangers of uncontrolled failures of the operating structures. Mathematical modeling of the failure is more preferred for the representation of the failure time as a function of many variables (4) or a function (8) of one variable.

As a example of the simplest case $N_Z = 1$, we will consider the homogeneous uniaxial tension rods failure due to a high-temperature creep. Such failure at a given temperature and a given tensile stress σ we will describe with the mathematical models based on Kachanov-Rabotnov phenomenological damage parameter and its kinetic equation with initial boundary condition, that can are represented as [4, 7, 8]

$$\frac{d\omega}{dt} = A\left(\frac{\sigma}{1-\omega}\right)^k, \ \omega(0) = 0 . \qquad (11)$$

Here A and k depend on the temperature characteristics of the material creep properties defined according to the experiments with specimens at tensile creep conditions. Equation (11) must be integrated up to the time of failure which is determined from the condition

$$\omega(t_*) = 1 . \qquad (12)$$

We integrate the equation (11) by parts considering the initial condition from (11) and the failure condition (12)

$$\int_0^1 (1-\omega)^k \, d\omega = \int_0^{t_*} A\sigma^k dt .$$

After evaluation of the integrals we obtain the analytical expression for the failure time depended on the given material properties presented in this example by A and k coefficients, and the external forces presented by the tensile stress σ

$$t_* = A^{-1}(k+1)^{-1}\sigma^{-k} . \qquad (13)$$

Let us consider the case of deterministic material properties and random stress σ. It is the simplest case with $N_Z = 1$, $z \equiv \sigma$, and with regard to the expression (13) reduced to the function (8) of the form (fig. 1)

$$t_*(z) = a \cdot z^{-k}, \ a = A^{-1}(k+1)^{-1} . \qquad (14)$$

We suppose that random values of stress σ are evenly distributed in the finite interval, so $\sigma_{min} \le \sigma \le \sigma_{max}$, i.e. the probability density function of the random variable z has the form (fig. 2)

$$\zeta(z) = \begin{cases} 1/(\sigma_{max} - \sigma_{min}), & z \in [\sigma_{min}, \sigma_{max}] \\ 0, & z \notin [\sigma_{min}, \sigma_{max}] \end{cases} . \qquad (15)$$

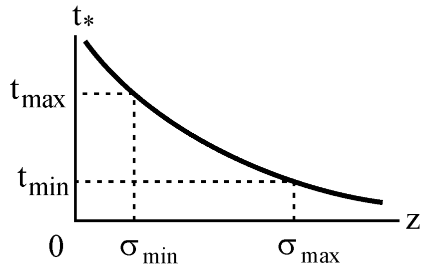

Fig.1. Depending between the failure time and the tensile stress of the rod.

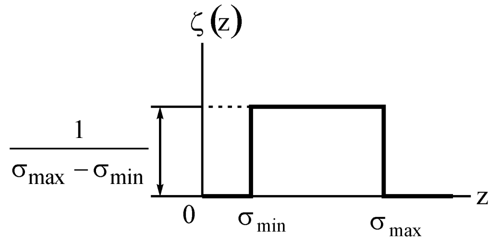

Fig.2. Probability density function of a random values of the tensile stress.

The probability density function $f = f(t_*)$ of the failure time values for the rod may be obtained from (10) for the form (14) of the failure time function (8) and the form (15) of the probability density function (9) of a random variable z as follows

$$f(t_*) = \begin{cases} \dfrac{a^{1/k}}{(\sigma_{max} - \sigma_{min})kt_*^{1+1/k}}, & t_* \in [t_{min}, t_{max}] \\ 0, & t_* \notin [t_{min}, t_{max}] \end{cases} . \qquad (16)$$

Here $t_{max} = t_*(\sigma_{min})$, $t_{min} = t_*(\sigma_{max})$ (fig. 2) and it is taken into account that the inverse function (14) has the form $z(t_*) = \left(a \cdot t_*^{-1}\right)^{1/k}$. In that way, the probability density function $f = f(t_*)$ of the failure time values for the rod is obtained in the form (16) and can be used for estimation of the durability indexes from (1).

3. The general approach to solution of the initial-boundary value problems of the creep theory for estimation of the failure time. The function (4) or (8) could be obtained by using the mathematical model of failure of the structural elements as deformable bodies.

The deformable body is considered as the volume $\Upsilon \subset R^3$ with the boundary surface υ, and each point of the body is defined by the position vector $\vec{r} \in \Upsilon$. We assume that the total deformations are small and each point of the body is in compliance with Lagrangian formalism for any instant time $t \geq 0$. The deformations are defined by the displacement vector depended on the time t and the position vector $\vec{r} \in \Upsilon$. Dispersion of the failure time is a consequence of dispersion of the material properties and external forces.

The main ideas of the method of operators [10] are used here to obtain the most generalized mathematical formulations of the problem, which are useful for building the computational algorithms for numerical solution of the problem. According to the method of operators let us introduce the vector $\mathbf{u}_1 = \mathbf{u}_1(\vec{r}, t)$ consisted of the parameters described the body as continuum, i.e. consisted of the components of the stress tensor σ_{ij}, strain tensor ε_{ij}, and the displacement vector u_i. We also introduce the $\mathbf{u}_2 = \mathbf{u}_2(\vec{r}, t)$ vector whose components are all the parameters described irreversible creep deformations of the body and a failure due to the creep, i.e. the components of the creep deformation tensor c_{ij} and the damage parameters [9]. The creep initial-boundary value problem for the body with above prescribed material properties and external forces $\mathbf{z} \in Z$ can be presented as follows [4, 5, 7, 8]

$$\mathbf{A}_1(\mathbf{z}) \cdot \mathbf{u}_1 + \mathbf{A}_2(\mathbf{z}) \cdot \mathbf{u}_2 = \mathbf{f}_1(\mathbf{u}_1; \mathbf{z}),\ \forall \vec{r} \in \Upsilon,$$
$$\mathbf{L}_1(\mathbf{z}) \cdot \mathbf{u}_1 = \mathbf{p}_1(\mathbf{z}) + \mathbf{L}_2(\mathbf{z}) \cdot \mathbf{u}_2,\ \forall \vec{r} \in \upsilon, \tag{17}$$

$$\frac{\partial \mathbf{u}_2}{\partial t} = \mathbf{f}_2(\mathbf{u}_2; \mathbf{u}_1; \mathbf{z}),\ \mathbf{u}_2\big|_{t=0} = \mathbf{0},\ \forall \vec{r} \in \Upsilon. \tag{18}$$

Here $\mathbf{A}_1(\mathbf{z})$, $\mathbf{A}_2(\mathbf{z})$ are the matrix operators and $\mathbf{f}_1(\mathbf{u}_1; \mathbf{u}_2; \mathbf{z})$ is the vector operator corresponded to differential equations; $\mathbf{L}_1(\mathbf{z})$, $\mathbf{L}_2(\mathbf{z})$ are matrix and $\mathbf{p}_1(\mathbf{z})$ is vector operator corresponded to the boundary conditions that determine the stress-strain state of the body at the creep deformations and damage parameters specified by vector $\mathbf{u}_2$, and material properties and external forces determined by $\mathbf{z}$. The vector operator $\mathbf{f}_2(\mathbf{u}_2; \mathbf{u}_1; \mathbf{z})$ corresponds to the creep-damage models used for the creep deformations and damage parameters velocities definition for a specified $\mathbf{u}_1$ and $\mathbf{z}$ vectors. Dependence of the operators $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{L}_1$, $\mathbf{L}_2$ and $\mathbf{f}_2$ on $\mathbf{z}$ is a consequence of the dependence of those operators on the material properties presented in $\mathbf{z}$; dependence of the $\mathbf{f}_1$ and $\mathbf{p}_1$ operators on $\mathbf{z}$ vector is a consequence of the dependence of those operators on external forces presented in $\mathbf{z}$.

Equations (17), (18) describe deformation of the body accounting for microscopic defects. Formation of the macroscopic defects is a consequence of accumulation of the microscopic defects and it can be determined by means of the conditions

$$\xi_1(\mathbf{u}_2) = 1,\ \xi_2(\mathbf{u}_2) = 1,\ \dots,\ \xi_{N_\xi}(\mathbf{u}_2) = 1, \tag{19}$$

where $\xi_k(\mathbf{u}_2)$, $k = 1, 2, \dots, N_\xi$ are the features of the specific mechanism of defect accumulation, so the values $\xi_k(\mathbf{u}_2) = 0$ correspond to the defects in an initial state, while the values $\xi_k(\mathbf{u}_2) = 1$ correspond to defects in a state prior to the formation of the macroscopic defect.

Equations (17), (18) must be integrated for a given vector $\mathbf{z}$ at one of conditions (19). In that way, accepting $\mathbf{u}_2 = \mathbf{u}_2(\vec{r}, t)$, the failure time t_* and location $\vec{r}_*$ of the macroscopic defect can be determined from the condition

$$\begin{gathered}\left(\xi_1(\vec{r}_*, t_*) = 1\right) \vee \left(\xi_2(\vec{r}_*, t_*) = 1\right) \vee \dots \\ \dots \vee \left(\xi_{N_\xi}(\vec{r}_*, t_*) = 1\right),\end{gathered} \tag{20}$$

where $\xi_k(\vec{r}, t) = \xi_k(\mathbf{u}_2(\vec{r}, t))$, $k = 1, 2, \dots, N_\xi$.

The initial-boundary value problem (17), (18) with the conditions (20) can be solved numerically for any given $\mathbf{z} \in Z$ vector as it was shown in [4, 5, 7, 8], and then we can define the failure time t_* corresponded to the $\mathbf{z} \in Z$ vector, i.e. as function in the form (4). The approach gives the table of the function (4) values obtained from numerical solution of the initial-boundary value problem (17), (18) with conditions (20) for various combinations of the z_i coordinates. We can construct the approximation of the function (4) based on the computed values of the function (4). The approximation can be even presented in the analytical form like (14), where a and k are the approximating coefficients.

4. Application of the proposed methods for estimations of the durability indexes of the pipes pressured under high-temperature creep conditions. As an example, an infinitely long cylinder with internal radius $r_1 = 10$ mm, external radius $r_2 = 15$ mm made from 18-8-type stainless steel and loaded by internal pressure p is considered. Because of the axial symmetry of the problem, the equations (17), (18) in the cylindrical coordinate system within the range $r_1 \leq r \leq r_2$ possess the following form [8]

$$\begin{gathered}-\frac{1}{E}\sigma_{rr} + \frac{\nu}{E}\sigma_{\theta\theta} + \frac{\partial u_r}{\partial r} - c_{rr} = 0, \\ -\frac{1}{E}\sigma_{\theta\theta} + \frac{\nu}{E}\sigma_{rr} + \frac{u_r}{r} - c_{\theta\theta} = 0, \\ \frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0, \\ \sigma_{rr}\big|_{r=r_1} = p,\ \sigma_{rr}\big|_{r=r_2} = 0\end{gathered}, \tag{21}$$

$$\frac{\partial c_{rr}}{\partial t} = \frac{3}{2}\frac{B\sigma_{eq}^{n-1}}{(1-\omega)^n}\left(\frac{2}{3}\sigma_{rr} - \frac{1}{3}\sigma_{\theta\theta}\right),$$

$$\frac{\partial c_{\theta\theta}}{\partial t} = \frac{3}{2}\frac{B\sigma_{eq}^{n-1}}{(1-\omega)^n}\left(\frac{2}{3}\sigma_{\theta\theta} - \frac{1}{3}\sigma_{rr}\right),$$

$$\frac{\partial\omega}{\partial t}=A\left(\frac{\sigma_{eq}}{1-\omega}\right)^k,\ \omega\big|_{t=0}=0,$$
$$c_{rr}\big|_{t=0}=0,\ c_{\theta\theta}\big|_{t=0}=0. \qquad (22)$$

Here $u_r=u_r(r,t)$ is the radial displacement; $\sigma_{rr}=\sigma_{rr}(r,t)$ and $\sigma_{\theta\theta}=\sigma_{\theta\theta}(r,t)$ are the radial and tangential stresses; $c_{rr}=c_{rr}(r,t)$ and $c_{\theta\theta}=c_{\theta\theta}(r,t)$ are radial and tangential creep deformations; $\omega=\omega(r,t)$ is the damage parameter; σ_{eq} is an equivalent stress equal to von Mises stresses intensity:

$$\sigma_{eq}=\frac{1}{\sqrt{2}}\sqrt{(\sigma_{\theta\theta}-\sigma_{rr})^2+\sigma_{rr}^2+\sigma_{\theta\theta}^2}.$$

In (21), (22) the materials properties at the given temperature $T=500°C$ of the 18-8-type stainless steel are presented by the Young modulus $E=1.62\cdot10^5$ MPa, Poisson's ratio $\nu=0.3$ and by the quantities $n=2.023$, $B=8.859\cdot10^{-13}\,MPa^{-n}/h$, $k=12.344$, $A=3.779\cdot10^{-33}\,MPa^{-k}/h$ that have been obtained by processing the isochronous creep curves and the long-term strength curves [8]. The failure time t_* and the location $\vec{r}_*$ of the macroscopic defect are determined from the condition

$$\omega(\vec{r}_*,t_*)=1. \qquad (23)$$

The equations (21), (22) together with (23) can be solved numerically by Galerkin-Bubnov and R-functions methods as proposed in [4, 5, 7, 8]. An accuracy of the approximate numerical solutions can be shown by the convergence of the solutions with increasing the number of the trial functions. Besides, the approximate solution corresponded to the elastic deformation at $t=0$ can be compared with the well-known Lame solution for thick-walled pressured cylinders. Here accuracy of the approximate solutions is not discussed, but some results of estimation of the durability indexes of the internal pressured pipes under high-temperature creep conditions will be discussed minutely.

Let us consider the case of the deterministic material properties and the random internal pressure p in the cylinder. The results of the numerical solution of the equations (21), (22) together with (23) show that the function (8) of the failure time t_* depending on the pressure p can be represented as approximation in the following form (fig. 3)

$$t_*(p)=a\cdot p^{-m},$$
$$a\cong6.924\cdot10^{31}\,h\cdot MPa^k,\ m\cong14.267. \qquad (24)$$

We suppose that the random values of p are evenly distributed in the finite interval and $p_{min}\le p\le p_{max}$, $p_{min}=62.5$ MPa, $p_{max}=67.5$ MPa, i.e. the probability density function of the pressure p has the form (fig. 4).

$$\zeta(p)=\begin{cases}1/(p_{max}-p_{min}), & p\in[p_{min},p_{max}]\\ 0, & p\notin[p_{min},p_{max}]\end{cases}. \qquad (25)$$

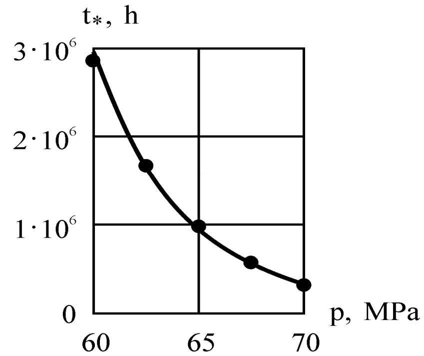

Fig.3. Approximation (curve) and computed points (markers) of the dependence between the failure time and the internal pressure in the cylinder.

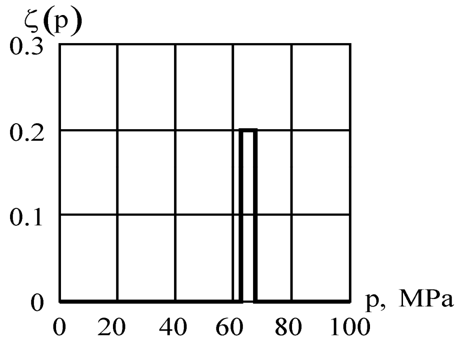

Fig.4. Probability density function of random values of the internal pressure in the cylinder.

The considered case of the cylinder pressured at the high-temperature creep conditions corresponds to $N_Z=1$, $z\equiv p$; using (10) and (24), (25) the problem can be reduced to the probability density function of the failure time values depicted in fig. 5. The durability indexes can be obtained using (1) and the probability density function. Thus we can compute the durability indexes such as the gamma-percentile life t_γ (fig. 6) and the mean life $\langle t_*\rangle$ of the cylinder as following

$$\langle t_*\rangle\cong9.959\cdot10^5\ h. \qquad (26)$$

The values of the gamma-percentile life t_γ (fig. 6) corresponded to the parameters $\gamma=75...95\%$ are less than the mean life value (26) obtained for the cylinder under the internal pressure and the high-temperature creep conditions. The values of the gamma-percentile life t_γ corresponded to the value $\gamma=75\%$ is $\cong72\%$ and is $\cong58\%$ in the case of the $\gamma=95\%$ value (fig. 6) of the mean life value (26). It is additionally found that the mean life $\langle t_*\rangle$ value (26) are in correspondence with the result for the gamma-percentile life t_γ obtained for the $\gamma\cong45\%$ value. In that way, the durability estimation based on the mean life value is inflated in the sense that it corresponds to the lower probability of the failure absence during the operation.

The gamma-percentile life is a more adequate assessment of the durability. It is possible to find the operating time corresponded to the allowable probability of the failure absence using the values of the gamma-percentile life. This is particularly important for reliable estimation of potential accidents with technical systems, which operation is permitted under the conditions of acceptable risk.

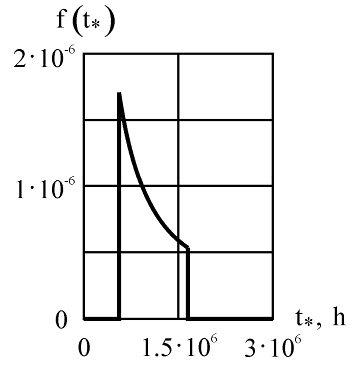

Fig.5. Probability density function of a random values of the failure times of the cylinder.

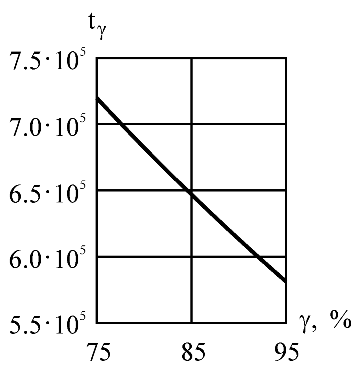

Fig.6. Gamma-percentile life of the cylinder under the internal pressure and the high-temperature creep conditions.

Conclusions. Estimation of the durability indexes of the elements of structures based on solution of the initial-boundary value problems of creep theory is proposed. The general approach to the estimations of the durability indexes in presence of dispersion of the operating conditions of the structural elements is shown. Estimation of the durability indexes is reduced to determination of the probably density of the failure time using the data on the operating conditions and their probably density functions. It is shown that dependence of the failure time on the operating conditions can be obtained by numerical solution of the initial-boundary value problems of the creep theory. The example of proposed method for estimations of the durability indexes of the internally pressured pipe under high-temperature creep conditions is considered in details. It is shown that estimation of durability of the structural elements basing on the mean life value is inflated in the sense that it corresponds to low probability of the failure absence during the operation time. The gamma-percentile life is a more adequate assessment of durability of the structural elements. Thus, it is possible to determine the operating time of the construction corresponded to the given allowable probability of the failure absence using the values of the gamma-percentile life.

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