МАТЕМАТИКА/MATHEMATICS

ON THE ASYMPTOTICS OF THE GENERALIZED FUP-FUNCTIONS

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In [1] V.A. Rvachev introduced the function

$$\mathrm{Fup}_n(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{itx}\left(\frac{\sin\frac{t}{2^{n+1}}}{\frac{t}{2^{n+1}}}\right)^{n+1}\prod_{j=n+2}^{\infty}\frac{\sin\frac{t}{2^j}}{\frac{t}{2^j}}dt,$$

$$n=0,1,2,\dots .$$

This function has various applications in such branches of mathematics as approximation theory [1, 2], wavelet theory [3] and mathematical modeling [4, 5]. Therefore the asymptotic behavior of $\mathrm{Fup}_n(x)$ as $n\to\infty$ is of interest.

In this paper we construct a generalization of the function $\mathrm{Fup}_n(x)$ and consider the problem of its asymptotics.

Consider the function

$$\mathrm{mup}_s(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{itx}F_s(t)dt\ ,\ F_s(t)=\prod_{k=1}^{\infty}\frac{\sin^2\frac{st}{(2s)^k}}{s^2\cdot\frac{t}{(2s)^k}\cdot\sin\frac{t}{(2s)^k}},$$

$$s\in\mathbb{N}$$

which is a solution with a compact support of the functional differential equation

$$y'(x)=2\cdot\sum_{k=1}^{s}\left(y(2sx+2s-2k+1)-y(2sx-2k+1)\right).$$

The function $\mathrm{mup}_s(x)$ was introduced by V.A. Rvachev and G.A. Starets in [6].

Let

$$f_{s,N,n}(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{itx}\left(\frac{\sin\frac{t}{N}}{\frac{t}{N}}\right)^{n+1}F_s\left(\frac{t}{N}\right)dt,$$

where $n,N\in\mathbb{N}$. It is obvious that this function is a generalization of the function $\mathrm{Fup}_n(x)$. Hence we will call it the generalized Fup-function.

Theorem 1. *For any $x\in\mathbb{R}$ it is true that*

$$\left|\frac{1}{N}f_{s,N,n}\left(\frac{x}{N}\right)-\frac{1}{\sqrt{2\pi\frac{n+1}{3}}}e^{-\frac{x^2}{2}}\right|\le\frac{2M+1}{\sqrt{(n+1)^3}}+\frac{0{,}9^{n-1}}{2}+\frac{1}{(n+1)e^{\frac{n+1}{6}}},$$

where $M=\int_{-1}^{1}x^2\mathrm{mup}_s(x)dx$.

So there exists an asymptote of generalized Fup-function and the first term of its asymptotic expansion is obtained.

Note that if $N=2(2s)^n$ then $f_{s,N,n}(x)$ equals to the function $\mathrm{Fmup}_s^{[n]}(x)$ which was introduced in [7].

Functions $\mathrm{Fup}_n(x)$ and $\mathrm{Fmup}_s^{[n]}(x)$ have "good" approximation properties.

Let $\tilde{W}_\infty^r$ be a class of functions $f\in C^r_{[-\pi,\pi]}$ such that $f^{(j)}(-\pi)=f^{(j)}(\pi)$ for any $j=0,1,\dots,r-1$ and $\left\|f^{(r)}\right\|_{C[-\pi,\pi]}\le 1$. Denote by $\tilde{W}_2^r$ a class of functions $f\in C^{r-1}_{[-\pi,\pi]}$ such that $f^{(j)}(-\pi)=f^{(j)}(\pi)$ for any $j=0,1,\dots,r-1$, $f^{(r-1)}(x)$ is absolutely continuous and $\left\|f^{(r)}\right\|_{L_2[-\pi,\pi]}\le 1$. Spaces of linear combinations of shifts of the function $\mathrm{Fup}_n(x)$ are asymptotically extremal for approximating $\tilde{W}_\infty^r$ in the norm of $C[-\pi,\pi]$ [1] and extremal for approximating $\tilde{W}_2^r$ in the norm of $L_2[-\pi,\pi]$ [2]. It was shown in [7] that spaces of linear combinations of shifts of the function $\mathrm{Fmup}_s^{[n]}(x)$ are asymptotically extremal for approximating functions from the classes $\tilde{W}_2^r$ in the norm of $L_2[-\pi,\pi]$. Moreover, $\mathrm{Fup}_n(x)$ and $\mathrm{Fmup}_s^{[n]}(x)$ are infinitely smooth and locally supported. Therefore these functions are convenient to use from the practical point of view and theorem 1 provides their usage for large n.

LITERATURE

1. Rvachev V.L., Rvachev V.A. Nonclassical methods of approximation theory in boundary value problem. – Kyiv, Naukova dumka, 1979. – 196 p. (in Russian)
2. Rvachev V.A. Compactly supported solutions of functional differential equations and their applications. // Russ.Math. Surveys. – 1990. – v. 45, N1. – P. 87–120.
3. BerkolaikoM.Z., NovikovI.Ya. On infinitely smooth compactly supported almost-wavelets. // Math. Notes. – 1994. – v. 56, N 3. – P. 877–883.
4. GotovacH., CvetkovicV., AndricevicR. Adaptive Fup multi-resolution approach to flow and advective transport in highly heterogeneous porous media: Methodology, accuracy and convergence. // Advances in Water Resources. – 2009. – v.32, N6. – P. 885–905.
5. GotovacH., AndricevicR., GotovacB. Multi-resolution adaptive modeling of groundwater flow and transport problems. // Advances in Water Resources. – 2007. – v. 30, N 5. – P. 1105–1126.
6. RvachevV.A., G.A. Starets. Some atomic functions and their applications. // Proc. Ukr. SSR Acad. Sci., Ser. A. – 1983. – N 11. – P. 22–24.
7. MakarichevV.A. Approximation of periodic functions using $\mathrm{mup}_s(x)$. // Math. Notes. – 2013. – v. 93, N6. – P. 858–880.

SOLUTION OF REAL LIFE STRUCTURAL-PARAMETRIC IDENTIFICATION PROBLEMS USING CONTINUED FRACTIONS

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Introduction. Although, system identification methods utilizing experimental data in a form of discrete-time series are widely used nowadays, there still not exists a well-developed universally recognized