МАТЕМАТИКА/MATHEMATICS

ON THE ASYMPTOTICS OF THE GENERALIZED FUP-FUNCTIONS

*Brysina I.V., *Makarichev V.A.*

N.Ye.ZhukovskyNationalAerospaceUniversity "KhAI", Kharkov, Ukraine

In [1] V.A. Rvachev introduced the function

$$\mathrm{Fup}_n(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{itx}\left(\frac{\sin\frac{t}{2^{n+1}}}{\frac{t}{2^{n+1}}}\right)^{n+1}\prod_{j=n+2}^{\infty}\frac{\sin\frac{t}{2^j}}{\frac{t}{2^j}}dt,$$

$$n = 0,1,2,\dots .$$

This function has various applications in such branches of mathematics as approximation theory [1, 2], wavelet theory [3] and mathematical modeling [4, 5]. Therefore the asymptotic behavior of $\mathrm{Fup}_n(x)$ as $n \to \infty$ is of interest.

In this paper we construct a generalization of the function $\mathrm{Fup}_n(x)$ and consider the problem of its asymptotics.

Consider the function

$$\mathrm{mup}_s(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{itx}F_s(t)dt\ ,\ F_s(t) = \prod_{k=1}^{\infty}\frac{\sin^2\frac{st}{(2s)^k}}{s^2\cdot\frac{t}{(2s)^k}\cdot\sin\frac{t}{(2s)^k}},$$

$$s \in \mathbb{N}$$

which is a solution with a compact support of the functional differential equation

$$y'(x) = 2\cdot\sum_{k=1}^{s}\big(y(2sx+2s-2k+1) - y(2sx-2k+1)\big).$$

The function $\mathrm{mup}_s(x)$ was introduced by V.A. Rvachev and G.A. Starets in [6].

Let

$$f_{s,N,n}(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{itx}\left(\frac{\sin\frac{t}{N}}{\frac{t}{N}}\right)^{n+1}F_s\left(\frac{t}{N}\right)dt,$$

where $n, N \in \mathbb{N}$. It is obvious that this function is a generalization of the function $\mathrm{Fup}_n(x)$. Hence we will call it the generalized Fup-function.

Theorem 1. *For any* $x \in \mathbb{R}$ *it is true that*

$$\left|\frac{1}{N}f_{s,N,n}\left(\frac{x}{N}\right) - \frac{1}{\sqrt{2\pi\frac{n+1}{3}}}e^{-\frac{x^2}{2}}\right| \le \frac{2M+1}{\sqrt{(n+1)^3}} + \frac{0{,}9^{n-1}}{2} + \frac{1}{(n+1)e^{\frac{n+1}{6}}},$$

where $M = \int_{-1}^{1} x^2\mathrm{mup}_s(x)dx$.

So there exists an asymptote of generalized Fup-function and the first term of its asymptotic expansion is obtained.

Note that if $N = 2(2s)^n$ then $f_{s,N,n}(x)$ equals to the function $\mathrm{Fmup}_s^{[n]}(x)$ which was introduced in [7].

Functions $\mathrm{Fup}_n(x)$ and $\mathrm{Fmup}_s^{[n]}(x)$ have "good" approximation properties.

Let $\tilde{W}_\infty^r$ be a class of functions $f \in C_{[-\pi,\pi]}^r$ such that $f^{(j)}(-\pi) = f^{(j)}(\pi)$ for any $j = 0,1,\dots,r-1$ and $\|f^{(r)}\|_{C[-\pi,\pi]} \le 1$. Denote by $\tilde{W}_2^r$ a class of functions $f \in C_{[-\pi,\pi]}^{r-1}$ such that $f^{(j)}(-\pi) = f^{(j)}(\pi)$ for any $j = 0,1,\dots,r-1$, $f^{(r-1)}(x)$ is absolutely continuous and $\|f^{(r)}\|_{L_2[-\pi,\pi]} \le 1$. Spaces of linear combinations of shifts of the function $\mathrm{Fup}_n(x)$ are asymptotically extremal for approximating $\tilde{W}_\infty^r$ in the norm of $C[-\pi,\pi]$ [1] and extremal for approximating $\tilde{W}_2^r$ in the norm of $L_2[-\pi,\pi]$ [2]. It was shown in [7] that spaces of linear combinations of shifts of the function $\mathrm{Fmup}_s^{[n]}(x)$ are asymptotically extremal for approximating functions from the classes $\tilde{W}_2^r$ in the norm of $L_2[-\pi,\pi]$. Moreover, $\mathrm{Fup}_n(x)$ and $\mathrm{Fmup}_s^{[n]}(x)$ are infinitely smooth and locally supported. Therefore these functions are convenient to use from the practical point of view and theorem 1 provides their usage for large n.

LITERATURE

1. Rvachev V.L., Rvachev V.A. Nonclassical methods of approximation theory in boundary value problem. – Kyiv, Naukova dumka, 1979. – 196 p. (in Russian)
2. Rvachev V.A. Compactly supported solutions of functional differential equations and their applications. // Russ.Math. Surveys. – 1990. – v. 45, N1. – P. 87–120.
3. BerkolaikoM.Z., NovikovI.Ya. On infinitely smooth compactly supported almost-wavelets. // Math. Notes. – 1994. – v. 56, N 3. – P. 877–883.
4. GotovacH., CvetkovicV., AndricevicR. Adaptive Fup multi-resolution approach to flow and advective transport in highly heterogeneous porous media: Methodology, accuracy and convergence. // Advances in Water Resources. – 2009. – v.32, N6. – P. 885–905.
5. GotovacH., AndricevicR., GotovacB. Multi-resolution adaptive modeling of groundwater flow and transport problems. // Advances in Water Resources. – 2007. – v. 30, N 5. – P. 1105–1126.
6. RvachevV.A., G.A. Starets. Some atomic functions and their applications. // Proc. Ukr. SSR Acad. Sci., Ser. A. – 1983. – N 11. – P. 22–24.
7. MakarichevV.A. Approximation of periodic functions using $\mathrm{mup}_s(x)$. // Math. Notes. – 2013. – v. 93, N6. – P. 858–880.

SOLUTION OF REAL LIFE STRUCTURAL-PARAMETRIC IDENTIFICATION PROBLEMS USING CONTINUED FRACTIONS

**Glushko O.V., Denishik S.S., Tkachov R.Yu.*

DonbassStateTechnicalUniversity, Alchevsk, Ukraine

Introduction. Although, system identification methods utilizing experimental data in a form of discrete-time series are widely used nowadays, there still not exists a well-developed universally recognized structural identification method neither for nonlinear dynamical systems nor for linear ones.

Identification approach using continued fractions which was proposed in [1] has many advantages such as possibility of simultaneous determination of both structure and parameters of a model, possibility of identification with different input signals in open-loop systems as well as in closed-loop systems, relative simplicity of algorithm realization with contemporary microprocessor devices… The approach shows perfect results in noise-free modeling but in real life tasks measurement noise and perturbations may lead to significant increase of model order that in many cases can make it impossible to identify object [1–4].

In this paper we propose a new approach to the SP-identification of time-delay systems that makes it possible to identify low-order models from noised measurements.

Main result. Assuming sampling time been constant, discrete-time experimental data can be represented in a form of formal Laurent power series:

$$f(z) = c_0 + c_1 z^{-1} + c_2 z^{-2} + c_3 z^{-3} + \ldots + c_m z^{-m}, \quad (1)$$

where $\{c_k\}_{k=0}^{m}$ – discrete sequence of output measurements, $\{c\} \in □$; $z = e^{Ts}$; T – sampling period, sec; $s = \sigma + j\omega$.

Then, on the basis of (1) a continued fraction can be formed. In this paper we'll use Rutishauser method of representing analytic functions by continued fractions (continued fractions are written in Rodgers notation):

$$f(z) = \frac{c_0}{1-}\,\frac{q_1^{(0)} z^{-1}}{1-}\,\frac{e_1^{(0)} z^{-1}}{1-}\,\frac{q_2^{(0)} z^{-1}}{1-}\,\frac{e_2^{(0)} z^{-1}}{1-}\ldots, \quad (2)$$

where $e_m^{(n)} \in □$; $q_m^{(n)} \in □$; $f(z) \in \hat{□}$; $\hat{□} = □ \cup [\infty]$; $\{e_m^{(n)}\}$, $\{q_m^{(n)}\}$ – sequences calculated with formulas: $e_m^{(n)} = q_m^{(n+1)} - q_m^{(n)} + q_{m-1}^{(n+1)}$; $q_{m+1}^{(n)} = e_m^{(n+1)} q_m^{(n+1)} / e_m^{(n)}$; m = 1,2,3…; n = 0,1,2,3…; $e_0^{(n)} = 0$; $q_1^{(n)} = c_{n+1} / c_n$.

To avoid zero division in (2) sequence $\{c_k\}_{k=0}^{m}$ is shifted to a first nonzero element. The first nonzero element and resulting continued fraction should be multiplied by z^{-d} according to the delay theorem, where d is a shift of a lattice function.

The determination of continued fraction coefficients can be realized by calculation of an identification matrix as it was realized in [2].

To determine low order model we have to formulate functional minimization problem with stability restrictions.

Let characteristic polynomial of a discrete transfer function is:

$$D(z) = a_0 + a_1 z + a_2 z^2 + \ldots + a_n z^n .$$

Then stability restrictions can be formulated in a form:

$$\begin{cases} |a_0| < a_n \\ |b_0| > |b_{n-1}| \\ |c_0| > |c_{n-2}| \\ \vdots \\ |r_0| > |r_2| \end{cases}, \quad (3)$$

where $b_k = \begin{vmatrix} a_0 & a_{n-k} \\ a_n & a_k \end{vmatrix}$, $c_k = \begin{vmatrix} b_0 & b_{n-1-k} \\ b_{n-1} & b_k \end{vmatrix}$, $d_k = \begin{vmatrix} c_0 & c_{n-2-k} \\ c_{n-2} & c_k \end{vmatrix}$, … .

System (3) should be supplemented with steady-state equation of model's transient process:

$$y_{st}(\infty) = \lim_{z\to 1}(z-1) f_{st}(z) = 0 \quad (4)$$

Having expressed (3) and (4) in terms of {c} determination of low order model can be realized through minimization of functional (5) taking into account restrictions (3,4).

$$J = \sum_{i=0}^{n} (\overline{c}_i - c_i)^2 , \quad (5)$$

where {c} – experimental data, $\{\overline{c}\}$ – variable parameters.

Then $\{\overline{c}\}$ is used for forming continued fraction that'll give us discrete transfer function of a model. Model in a form of continuous transfer function can be derived with matched Z-transform as it was realized in [2, 3].

LITERATURE

1. Карташов В.Я. Эквивалентность дискретных моделей – реальность? // Промышленные АСУ и контроллеры. – 2006. – № 8. – С. 40–44.
2. GlushkoO.V.,Tkachev R.Yu. On structural-parametric identification of time-delay systems from real impulse response // Contemporaryproblemsofmathematics, mechanicsandcomputingsciences.– Kharkov. 2012. (accepted)
3. Ткачев Р.Ю., Глушко О.В. Структурно-параметрическая идентификация объектов с рециклом на основе дискретной последовательности выходной координаты // Збірник наукових праць ДонДТУ.– 2012.– № 36.– С. 415–425.
4. Карташов В.Я., Новосельцева М.А. Структурно-параметрическая идентификация линейных стохастических объектов с использованием непрерывных дробей // Управление большими системами.– 2008.– № 21.– С. 27–48.

EXPONENTIAL DIVISOR FUNCTIONS

Lelechenko A. V.

Odessa National University, Odessa, Ukraine

Consider a set of arithmetic functions $\mathcal{A}$, a set of multiplicative prime-independent functions $\mathcal{M}_{PI}$ and operator $E : \mathcal{A} \to \mathcal{M}_{PI}$, which is defined as $(Ef)(p^a) = f(a)$. The behaviour of Ef for various special cases of f has been widely studied, starting with the pioneering paper of Subbarao [1] on $E\tau$ and $E\mu$.

Our first aim is to investigate asymptotic properties of multiple applications of E on arithmetic functions. Our second aim is to study $E^m f$ over squarefull and cubefull numbers.