structural identification method neither for nonlinear dynamical systems nor for linear ones.

Identification approach using continued fractions which was proposed in [1] has many advantages such as possibility of simultaneous determination of both structure and parameters of a model, possibility of identification with different input signals in open-loop systems as well as in closed-loop systems, relative simplicity of algorithm realization with contemporary microprocessor devices… The approach shows perfect results in noise-free modeling but in real life tasks measurement noise and perturbations may lead to significant increase of model order that in many cases can make it impossible to identify object [1–4].

In this paper we propose a new approach to the SP-identification of time-delay systems that makes it possible to identify low-order models from noised measurements.

Main result. Assuming sampling time been constant, discrete-time experimental data can be represented in a form of formal Laurent power series:

$$f(z) = c_0 + c_1 z^{-1} + c_2 z^{-2} + c_3 z^{-3} + ... + c_m z^{-m} \ ,(1)$$

where $\{c_k\}_{k=0}^{m}$ – discrete sequence of output measurements, $\{c\} \in \mathbb{C}$; $z = e^{Ts}$; T – sampling period, sec; $s = \sigma + j\omega$.

Then, on the basis of (1) a continued fraction can be formed. In this paper we'll use Rutishauser method of representing analytic functions by continued fractions (continued fractions are written in Rodgers notation):

$$f(z) = \frac{c_0}{1\ -}\ \frac{q_1^{(0)} z^{-1}}{1\ -}\ \frac{e_1^{(0)} z^{-1}}{1\ -}\ \frac{q_2^{(0)} z^{-1}}{1\ -}\ \frac{e_2^{(0)} z^{-1}}{1\ -}\ ... ,(2)$$

where $e_m^{(n)} \in \mathbb{C}$; $q_m^{(n)} \in \mathbb{C}$; $f(z) \in \hat{\mathbb{C}}$; $\hat{\mathbb{C}} = \mathbb{C} \cup [\infty]$; $\{e_m^{(n)}\}$, $\{q_m^{(n)}\}$ – sequences calculated with formulas: $e_m^{(n)} = q_m^{(n+1)} - q_m^{(n)} + q_{m-1}^{(n+1)}$; $q_{m+1}^{(n)} = e_m^{(n+1)} q_m^{(n+1)} / e_m^{(n)}$; m = 1,2,3…; n = 0,1,2,3…; $e_0^{(n)} = 0$; $q_1^{(n)} = c_{n+1} / c_n$.

To avoid zero division in (2) sequence $\{c_k\}_{k=0}^{m}$ is shifted to a first nonzero element. The first nonzero element and resulting continued fraction should be multiplied by z^{-d} according to the delay theorem, where d is a shift of a lattice function.

The determination of continued fraction coefficients can be realized by calculation of an identification matrix as it was realized in [2].

To determine low order model we have to formulate functional minimization problem with stability restrictions.

Let characteristic polynomial of a discrete transfer function is:

$$D(z) = a_0 + a_1 z + a_2 z^2 + ... + a_n z^n .$$

Then stability restrictions can be formulated in a form:

$$\begin{cases} |a_0| < a_n \\ |b_0| > |b_{n-1}| \\ |c_0| > |c_{n-2}| \\ \vdots \\ |r_0| > |r_2| \end{cases} , \qquad (3)$$

where $b_k = \begin{vmatrix} a_0 & a_{n-k} \\ a_n & a_k \end{vmatrix}$, $c_k = \begin{vmatrix} b_0 & b_{n-1-k} \\ b_{n-1} & b_k \end{vmatrix}$, $d_k = \begin{vmatrix} c_0 & c_{n-2-k} \\ c_{n-2} & c_k \end{vmatrix}$, … .

System (3) should be supplemented with steady-state equation of model's transient process:

$$y_{st}(\infty) = \lim_{z \to 1} (z-1) f_{st}(z) = 0 \ (4)$$

Having expressed (3) and (4) in terms of {c} determination of low order model can be realized through minimization of functional (5) taking into account restrictions (3,4).

$$J = \sum_{i=0}^{n} (\overline{c}_i - c_i)^2 \ , \qquad (5)$$

where {c} – experimental data, $\{\overline{c}\}$ – variable parameters.

Then $\{\overline{c}\}$ is used for forming continued fraction that'll give us discrete transfer function of a model. Model in a form of continuous transfer function can be derived with matched Z-transform as it was realized in [2, 3].

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EXPONENTIAL DIVISOR FUNCTIONS

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Consider a set of arithmetic functions $\mathcal{A}$, a set of multiplicative prime-independent functions $\mathcal{M}_{PI}$ and operator $E : \mathcal{A} \to \mathcal{M}_{PI}$, which is defined as $(Ef)(p^a) = f(a)$. The behaviour of Ef for various special cases of f has been widely studied, starting with the pioneering paper of Subbarao [1] on $E\tau$ and $E\mu$.

Our first aim is to investigate asymptotic properties of multiple applications of E on arithmetic functions. Our second aim is to study $E^m f$ over squarefull and cubefull numbers.

Let $\gamma(0)=2$, $\gamma(m)=2^{\gamma(m-1)}$; let also $\tau(n)=\sum_{d|n}1$. Denote

$$\tau(a_1,\ldots,a_k;n)=\sum_{d_1^{a_1}\cdots d_k^{a_k}=n}1,$$

and let $\Delta(\cdots;x)$ be an error term in the asymptotic estimate of the sum $\sum_{n\le x}\tau(\cdots;n)$. Further, $\theta(\cdots)$ denotes a real value such that $\Delta(\cdots;x)\ll x^{\theta(\cdots)+\varepsilon}$.

Theorem 1. *For a fixed integer* $m>1$ *we have*

$$\sum_{n\le x}E^m\tau(n)=A_m x+B_m x^{1/\gamma(m)}+R_m(x),$$

where A_m *and* B_m *are computable constants,* $x^{1/2(\gamma(m)+1)}\ll R_m(x)\ll x^{\alpha_m+\varepsilon}$. *Also under Riemann hypothesis*

$$\alpha_m=\frac{1-\theta(1,\gamma(m))}{\gamma(m)+2-2(\gamma(m)+1)\theta(1,\gamma(m))}.$$

We get precise unconditional estimates of α_m as a result of the following lemma.

Lemma 2. *For a fixed integer* $r\ge 5$

$$\theta(1,2^r)=\frac{2^r-2r}{2^{2r}-r\cdot 2^r-2r^2+2r-4}<\frac{1}{2^r+r}.$$

Further, abbreviate $a\times k$ for a sequence of k arguments $a,\ldots,a$. We obtain an analog of Theorem 1 on asymptotic properties of $E^m\tau(1\times k;\cdot)$ and improve Tóth's theorem **Ошибка! Источник ссылки не найден.** showing that

$$\theta(1,1\times(k-1))=\frac{1}{1+1-\theta(1\times(k-1))}.$$

In the case of $E^m\tau(1\times 3;\cdot)$ sharper estimates can be given. To achieve this goal we have proved following lemmas.

Lemma 3. *For a fixed integer* $r\ge 10$ *we have*

$$\theta(1,2^r,2^r)=\frac{26\cdot 2^{2r}-(29r+41)2^r+16r^2+12r+32}{26\cdot 2^{3r}-(16r+41)2^{2r}+(24r-3)2^r+16r+12}<\frac{1}{2^r+1}.$$

Lemma 4. *Consider a multiplicative function* f *such that*

$$\sum_{n=1}^{\infty}f(n)n^s=\frac{\zeta(as)\zeta^r(bs)}{\zeta^k(cs)}:=F(s),$$

where $2a\le b<c<2(a+b)$. Let $\Delta(x)$ be defined implicitly by the equation

$$S(x):=\sum_{n\le x}f(n)=\left(\operatorname*{res}_{s=1/a}+\operatorname*{res}_{s=1/b}\right)F(s)x^s s^{-1}+\Delta(x).$$

Then under RH for any $1\le y\le x^{1/c}$

$$\Delta(x)=\sum_{l\le y}\mu_k(l)\Delta(a,b\times r;x/l^c)+O(x^{1/2a+\varepsilon}y^{1/2-c/2a}+x^{\varepsilon}),$$

where μ_k is a multiplicative function such that $\sum_{n=1}^{\infty}\mu_k(n)n^{-s}=\zeta^{-k}(s)$.

We also investigate properties of $E^m f$ for arbitrary arithmetic function f and large enough m. We show that they can be estimated in the quite similar manner as $E^m\tau(1\times k;\cdot)$ was.

One can also learn $E^m f$ over squarefull and cubefull numbers, see [3]. This kind of estimates deeply depends on higher moments of Riemann zeta-function for a given $\sigma=\operatorname{Re}s$. We developed an algorithm of minimizing of objective function over exponent pairs under linear constrains. This algorithm allows us to obtain easily good pointwise estimates for zeta-function's moments, improving interval estimates from [4] and, thus, improves the results of [3].

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SOME PROPERTIES OF STRONGLY–PRIME MODULES

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Let R be associative ring with $1\ne 0$. Left ideal p of the ring R is called prime, if for every $x,y\in R$, $xRy\subseteq p$ implies either $x\in p$ or $y\in p$. Clearly, left prime ideal is two-sided iff it is prime in classical way. Set of all two-sided prime ideals is denoted by $\mathrm{Spec}(R)$ and is called (prime) spectrum of ring R. The space $\mathrm{spec}(R)$ may by defined in another way: it is the set of all strongly prime left ideals. Recall that left ideal p of the ring R is called strongly-prime, if for every $x\in R\setminus p$ there exist finite set V of the ring R, that $(p:Vx)=\{r\in R: rVx\subseteq p\}\subseteq p$. Clearly, every strongly-prime left ideal of ring R is prime left ideal and every maximal left ideal is strongly-prime. Nonzeroleftmodule M over ring R is called strongly-prime, if for any nonzero $x,y\in M$ there exist finite subset $\{a_1,a_2,\cdots,a_n\}\subseteq R$, that $\mathrm{Ann}_R\{a_1x,a_2x,\cdots,a_nx\}\subseteq$, $\subseteq\mathrm{Ann}_R\{y\}$ $(ra_1x=ra_2x=\ldots=ra_nx=0), r\in R$ implies $ry=0$ [1]. Submodule p of some module M is called strongly-prime, if quotient module M/p is strongly-prime R-module. The set of all strongly-prime submodules of module M is called left prime spectrum of M and is denoted by $\mathrm{spec}(M)$.

Lemma 1. *Let* p *and* q *are strongly-prime modules. Then such properties are hold:*

1. *If* $p \approx q$ *, than* $p \cap q$ *is strongly-prime module and* $p \approx p \cap q$;

2. *If* $p \cap q$ *is strongly-prime module, then either* $p \subseteq q$ *or* $p \supseteq q$ *or* $p \approx q$ *;*