Let $\gamma(0)=2$, $\gamma(m)=2^{\gamma(m-1)}$; let also $\tau(n)=\sum_{d|n}1$. Denote

$$\tau(a_1,\ldots,a_k;n)=\sum_{d_1^{a_1}\cdots d_k^{a_k}=n}1,$$

and let $\Delta(\cdots;x)$ be an error term in the asymptotic estimate of the sum $\sum_{n\le x}\tau(\cdots;n)$. Further, $\theta(\cdots)$ denotes a real value such that $\Delta(\cdots;x)\ll x^{\theta(\cdots)+\varepsilon}$.

Theorem 1. *For a fixed integer* $m>1$ *we have*

$$\sum_{n\le x}E^m\tau(n)=A_m x+B_m x^{1/\gamma(m)}+R_m(x),$$

where A_m *and* B_m *are computable constants,* $x^{1/2(\gamma(m)+1)}\ll R_m(x)\ll x^{\alpha_m+\varepsilon}$. *Also under Riemann hypothesis*

$$\alpha_m=\frac{1-\theta(1,\gamma(m))}{\gamma(m)+2-2(\gamma(m)+1)\theta(1,\gamma(m))}.$$

We get precise unconditional estimates of α_m as a result of the following lemma.

Lemma 2. *For a fixed integer* $r\ge 5$

$$\theta(1,2^r)=\frac{2^r-2r}{2^{2r}-r\cdot 2^r-2r^2+2r-4}<\frac{1}{2^r+r}.$$

Further, abbreviate $a\times k$ for a sequence of k arguments $a,\ldots,a$. We obtain an analog of Theorem 1 on asymptotic properties of $E^m\tau(1\times k;\cdot)$ and improve Tóth's theorem **Ошибка! Источник ссылки не найден.** showing that

$$\theta(1,1\times(k-1))=\frac{1}{1+1-\theta(1\times(k-1))}.$$

In the case of $E^m\tau(1\times 3;\cdot)$ sharper estimates can be given. To achieve this goal we have proved following lemmas.

Lemma 3. *For a fixed integer* $r\ge 10$ *we have*

$$\theta(1,2^r,2^r)=\frac{26\cdot 2^{2r}-(29r+41)2^r+16r^2+12r+32}{26\cdot 2^{3r}-(16r+41)2^{2r}+(24r-3)2^r+16r+12}<\frac{1}{2^r+1}.$$

Lemma 4. *Consider a multiplicative function* f *such that*

$$\sum_{n=1}^{\infty}f(n)n^s=\frac{\zeta(as)\zeta^r(bs)}{\zeta^k(cs)}:=F(s),$$

where $2a\le b<c<2(a+b)$. Let $\Delta(x)$ be defined implicitly by the equation

$$S(x):=\sum_{n\le x}f(n)=\left(\operatorname*{res}_{s=1/a}+\operatorname*{res}_{s=1/b}\right)F(s)x^s s^{-1}+\Delta(x).$$

Then under RH for any $1\le y\le x^{1/c}$

$$\Delta(x)=\sum_{l\le y}\mu_k(l)\Delta(a,b\times r;x/l^c)+O(x^{1/2a+\varepsilon}y^{1/2-c/2a}+x^{\varepsilon}),$$

where μ_k is a multiplicative function such that $\sum_{n=1}^{\infty}\mu_k(n)n^{-s}=\zeta^{-k}(s)$.

We also investigate properties of $E^m f$ for arbitrary arithmetic function f and large enough m. We show that they can be estimated in the quite similar manner as $E^m\tau(1\times k;\cdot)$ was.

One can also learn $E^m f$ over squarefull and cubefull numbers, see [3]. This kind of estimates deeply depends on higher moments of Riemann zeta-function for a given $\sigma=\operatorname{Re}s$. We developed an algorithm of minimizing of objective function over exponent pairs under linear constrains. This algorithm allows us to obtain easily good pointwise estimates for zeta-function's moments, improving interval estimates from [4] and, thus, improves the results of [3].

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SOME PROPERTIES OF STRONGLY–PRIME MODULES

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Let R be associative ring with $1\ne 0$. Left ideal p of the ring R is called prime, if for every $x,y\in R$, $xRy\subseteq p$ implies either $x\in p$ or $y\in p$. Clearly, left prime ideal is two-sided iff it is prime in classical way. Set of all two-sided prime ideals is denoted by $\operatorname{Spec}(R)$ and is called (prime) spectrum of ring R. The space $\operatorname{spec}(R)$ may by defined in another way: it is the set of all strongly prime left ideals. Recall that left ideal p of the ring R is called strongly-prime, if for every $x\in R\setminus p$ there exist finite set V of the ring R, that $(p:Vx)=\{r\in R: rVx\subseteq p\}\subseteq p$. Clearly, every strongly-prime left ideal of ring R is prime left ideal and every maximal left ideal is strongly-prime. Nonzeroleftmodule M over ring R is called strongly-prime, if for any nonzero $x,y\in M$ there exist finite subset $\{a_1,a_2,\cdots,a_n\}\subseteq R$, that $\operatorname{Ann}_R\{a_1x,a_2x,\cdots,a_nx\}\subseteq$, $\subseteq\operatorname{Ann}_R\{y\}$ $(ra_1x=ra_2x=\ldots=ra_nx=0), r\in R$ implies $ry=0$ [1]. Submodule p of some module M is called strongly-prime, if quotient module M/p is strongly-prime R-module. The set of all strongly-prime submodules of module M is called left prime spectrum of M and is denoted by $\operatorname{spec}(M)$.

Lemma 1. *Let* p *and* q *are strongly-prime modules. Then such properties are hold:*

1. If $p\approx q$, *than* $p\cap q$ *is strongly-prime module and* $p\approx p\cap q$;

2. If $p\cap q$ *is strongly-prime module, then either* $p\subseteq q$ *or* $p\supseteq q$ *or* $p\approx q$;

Lemma 2. *Let* $p_1, \ldots p_n$ *be finite family of such strongly-prime modules, that* $p_1 \approx \ldots \approx p_n$, *than* $\cap_{i=1}^{n} p_i$ *is strongly-prime module and* $p_1 \approx \cap_{i=1}^{n} p_i$.

Lemma 3. *For every strongly-prime left submodule* p *of some module* M *holds that* $b(p) \in Cspec(M)$ [2].

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ON CLASSICAL DIFFERENTIALLY PRIMARY SUBMODULES

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Commutative differential rings with nonzero identity and the set $\Delta = \{\delta_1, ..., \delta_n\}$ of paiwise commutative derications, and unitary modules over them are considered [1].

A proper differential submodule Q of the differential module M is called *classical differentially primary submodule* of M if for each $a, b \in R$, $n, k \in \mathbb{N} \cup \{0\}$, and a differential submodule $N \subseteq M$, $ab^{(n)}N \subseteq Q$ follows either $a^{(n)}N \subseteq Q$ or $b^k N \subseteq Q$.

Theorem 1.*Let* M *be a Noetherian differential* R*–module and* Q *be its proper submodule. Then* Q *is a classical differentially primary submodule if and only if* $(Q:N)$ *is a differentially primary ideal of* R *for every differential submodule* N *of* M *such that* $N \not\subset Q$.

Theorem 2.*Let* M *be a differential* R*-module and* Q *be its proper submodule. Then the following statements are equivalent:*

1. Q *is a classical differentially primary submodule;*

2. Forevery $m \in M$, $(Q : m^{(\infty)})$ *is a differentially primary ideal of* R.

An R-module M is called *differentially multiplication* [2] if for every differential submodule N of M there exists a differential ideal I of R such that $N = IM$.

Theorem 3.*Let* M *be a differentially multiplication* R*-module and* Q *be its proper submodule. Then the following statements are equivalent:*

1. Q *is a classical differentially primary submodule;*

2. Q *is a differentially primary submodule;*

3. $(Q:M)$ *is a differentially primary ideal of* R*;*

4. $Q = QM$, *where* Q *is a differentially primary ideal which is maximal with respect to the property* $IM = Q$ *follows* $I \subseteq Q$ (*I is a differential ideal of* R).

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SUBHARMONIC FUNCTIONS IN A UNIT BALL THAT GROWS NEAR A PART OF THE BOUNDARY

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We consider subharmonic functions in the unit n-dimensional ball, which grow near some part of the boundary, and get integral estimates for their Riesz measures.

To formulate our results we need some notations. Suppose B is n-dimensional ball, $E = \overline{E} \subset \partial B, \phi : R^+ \to R^+$ is monotonically decreasing continues function, $\phi(t) \to \infty$ as $t \to 0$. For $z \in \overline{B}$ put $\rho(z) = dist(z, E)$, $F(t) = m_{n-1}\{\zeta \in \partial B : \rho(\zeta) < t\}$, where m_{n-1} is the normalized (n−1)-dimensional Lebesgue measure.

Theorem 1. *Let v(z) be a subharmonic function in B,* $v(z) \neq -\infty$,

$$v(z) \le \phi(\rho(z)) \quad (1)$$

for all $z \in B$. *If*

$$\int_0^2 \phi(s) dF(s) < \infty \quad (2)$$

then the Reisz measure $\mu = c_{n-1}\Delta v$ *satisfies the condition*

$$\int_B (1 - |\lambda|)\mu(d\lambda) < \infty. \quad (3)$$

When condition (2) is broken, integral (3) may diverge. In this case we can control the growth of Riesz measure as well.

Theorem 2.*Suppose* ϕ, ψ *are absolutely continues positive functions on (0, 2),* $\phi(t)$ *monotonically decreases and* $\psi(t)$ *monotonically increases,* $\phi(t) \to +\infty$ *as* $t \to +0$, $\psi(t) \to 0$ *as* $t \to +0$, *and*

$$\int_0^1 (-\phi'(t))\psi(t)F(t)dt < \infty \quad (4)$$

If condition (1) is satisfied for a subharmonic function v(z) in B, then its Riesz measure μ satisfies the condition

$$\int_B \psi(k_n \rho(\lambda))(1 - |\lambda|)\mu(d\lambda) < \infty \quad (5)$$

where the constant $k_n \le 1$ *depends on the dimension only.*

Also, we show that our results are close to optimal ones.