

**Lemma 2.** Let  $p_1, \dots, p_n$  be finite family of such strongly-prime modules, that  $p_1 \approx \dots \approx p_n$ , than  $\bigcap_{i=1}^n p_i$  is strongly-prime module and  $p_1 \approx \bigcap_{i=1}^n p_i$ .

**Lemma 3.** For every strongly-prime left submodule  $p$  of some module  $M$  holds that  $b(p) \in \text{Cspec}(M)$  [2].

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### ON CLASSICAL DIFFERENTIALLY PRIMARY SUBMODULES

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Commutative differential rings with nonzero identity and the set  $\Delta = \{\delta_1, \dots, \delta_n\}$  of pairwise commutative derivations, and unitary modules over them are considered [1].

A proper differential submodule  $Q$  of the differential module  $M$  is called *classical differentially primary submodule* of  $M$  if for each  $a, b \in R$ ,  $n, k \in \mathbb{N} \cup \{0\}$ , and a differential submodule  $N \subseteq M$ ,  $ab^{(n)}N \subseteq Q$  follows either  $a^{(n)}N \subseteq Q$  or  $b^k N \subseteq Q$ .

**Theorem 1.** Let  $M$  be a Noetherian differential  $R$ -module and  $Q$  be its proper submodule. Then  $Q$  is a classical differentially primary submodule if and only if  $(Q : N)$  is a differentially primary ideal of  $R$  for every differential submodule  $N$  of  $M$  such that  $N \not\subseteq Q$ .

**Theorem 2.** Let  $M$  be a differential  $R$ -module and  $Q$  be its proper submodule. Then the following statements are equivalent:

1.  $Q$  is a classical differentially primary submodule;
2. For every  $m \in M$ ,  $(Q : m^{(\infty)})$  is a differentially primary ideal of  $R$ .

An  $R$ -module  $M$  is called *differentially multiplication* [2] if for every differential submodule  $N$  of  $M$  there exists a differential ideal  $I$  of  $R$  such that  $N = IM$ .

**Theorem 3.** Let  $M$  be a differentially multiplication  $R$ -module and  $Q$  be its proper submodule. Then the following statements are equivalent:

1.  $Q$  is a classical differentially primary submodule;
2.  $Q$  is a differentially primary submodule;
3.  $(Q : M)$  is a differentially primary ideal of  $R$ ;
4.  $Q = QM$ , where  $Q$  is a differentially primary ideal which is maximal with respect to the property  $IM = Q$  follows  $I \subseteq Q$  ( $I$  is a differential ideal of  $R$ ).

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### SUBHARMONIC FUNCTIONS IN A UNIT BALL THAT GROWS NEAR A PART OF THE BOUNDARY

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We consider subharmonic functions in the unit  $n$ -dimensional ball, which grow near some part of the boundary, and get integral estimates for their Riesz measures.

To formulate our results we need some notations. Suppose  $B$  is  $n$ -dimensional ball,  $E = \bar{E} \subset \partial B, \phi : R^+ \rightarrow R^+$  is monotonically decreasing continues function,  $\phi(t) \rightarrow \infty$  as  $t \rightarrow 0$ . For  $z \in \bar{B}$  put  $\rho(z) = \text{dist}(z, E)$ ,  $F(t) = m_{n-1} \{ \zeta \in \partial B : \rho(\zeta) < t \}$ , where  $m_{n-1}$  is the normalized  $(n-1)$ -dimensional Lebesgue measure.

**Theorem 1.** Let  $v(z)$  be a subharmonic function in  $B$ ,  $v(z) \neq -\infty$ ,

$$v(z) \leq \phi(\rho(z)) \quad (1)$$

for all  $z \in B$ . If

$$\int_0^2 \phi(s) dF(s) < \infty \quad (2)$$

then the Riesz measure  $\mu = c_{n-1} \Delta v$  satisfies the condition

$$\int_B (1 - |\lambda|) \mu(d\lambda) < \infty. \quad (3)$$

When condition (2) is broken, integral (3) may diverge. In this case we can control the growth of Riesz measure as well.

**Theorem 2.** Suppose  $\phi, \psi$  are absolutely continues positive functions on  $(0, 2)$ ,  $\phi(t)$  monotonically decreases and  $\psi(t)$  monotonically increases,  $\phi(t) \rightarrow +\infty$  as  $t \rightarrow +0$ ,  $\psi(t) \rightarrow 0$  as  $t \rightarrow +0$ , and

$$\int_0^1 (-\phi'(t)) \psi(t) F(t) dt < \infty \quad (4)$$

If condition (1) is satisfied for a subharmonic function  $v(z)$  in  $B$ , then its Riesz measure  $\mu$  satisfies the condition

$$\int_B \psi(k_n \rho(\lambda)) (1 - |\lambda|) \mu(d\lambda) < \infty \quad (5)$$

where the constant  $k_n \leq 1$  depends on the dimension only.

Also, we show that our results are close to optimal ones.