ALGORITHMS OF GENERATION OF ARTERIAL VASCULATURES THAT FILL IN A GIVEN VOLUME

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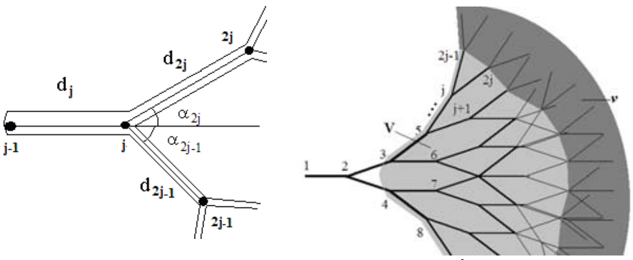
Blood vessel vasculatures, bronchial trees, trophic fluid flow systems in animals, phloem and xylem conducting systems in plants provide fluid delivery from one point to a distributed system of 'customers' or live cells. The transportation systems are presented by branching trees of tubes with elastic, rigid or porous walls. Mostly the tubes form bifurcations when a parent vessel is divided into two daughter vessels with smaller diameters. In the case the transportation system can be considered as binary tree. In other cases the trifurcations or more complex systems with internal cycles are also present. The branching systems of tubes fill in a given volume (internal organ) so that in the small vicinity of every cell there is a tube of the maximal branching order. Such systems are called space-filling trees [1] and generation of the branching systems filling in a volume without internal cross sections is important for development of detailed digital models of the cardiovascular and respiratory systems of humans and animals [2] as well as digital models of plants [3].

Since three tubes forming arterial bifurcations are mostly located in the same plane (fig.1a), the trees could be generated by small space rotation of the planes of bifurcations of two daughter vessels in relation to the plane of bifurcation of the parent vessel [4].



a b

Fig.1. Plane bifurcation of three tubes (a) and a binary tree filling in a given volume (b)

Another algorithm [5] is based on the well-known relationships between the diameters d_j, d_{2j-1}, d_{2j} and branching angles $\alpha_{2j-1}, \alpha_{2j}$ (fig.1a)

$$(d_j)^{\gamma} = (d_{2j-1})^{\gamma} + (d_{2j})^{\gamma} \quad (1)$$

$$\cos\alpha_{2j-1} = \frac{d_j^4 + d_{2j-1}^4 - d_{2j}^4}{2d_j^2 d_{2j-1}^2}, \cos\alpha_{2j} = \frac{d_j^4 - d_{2j-1}^4 + d_{2j}^4}{2d_{2j-1}^2 d_{2j}^2}. \quad (2)$$

The arterial, venous, airway bifurcations satisfy (1), (2) with high accuracy, while the relationships between the diameters and lengthes $L_j = L_j(d_j)$ based on different measurement data demonstrate quite big dispersion. As it was shown in [5], the diameters of the whole binary tree could be generated on (1) starting from the known diameter of the first (feeding) tube. Then all the branching angles could be computed on (2). If one assumes a uniform distribution of the smallest (capillary) vessels in some volume v (marked by dark grey in fig.1b), the values of the branching angles in the bifurcations of the maximal branching order will determine the coordinates of the bifurcation points and, in this way, the lengthes of the vessels. In [5] v was treated as plane layer, and here the same problem is solved for v as relatively thin layer located over a smooth surface of arbitrary curvature S.

Let's assume the coordinates of the outlets of the last order tubes $n = 4j, 4j-1, 4j-2, 4j-3$ (fig.2) are uniformly distributed over S according to localization of the capillary vessels in v. Then the angles $\alpha_{4j-1} + \alpha_{4j}$ and $\alpha_{4j-3} + \alpha_{4j-2}$ are determined by (2) and the bifurcation points $n = 2j-1, 2j$ are located on the corresponding circles with radii

$$R_{2j-1} = \frac{l_{4j-3,4j-2}}{2\cos(\alpha_{4j-2} + \alpha_{4j-3})}, R_{2j} = \frac{l_{4j-1,4j}}{2\cos(\alpha_{4j-1} + \alpha_{4j})},$$

where $l_{j,k}$ is the distance between the points $n = j$ and $n = k$.

Location of the points $n = 2j-1, 2j$ on the circles are determined by location of the point $n = j$ which defines locations of the tubes L_{2j-1}, L_{2j} and, therefore, the axis separated the angles $\alpha_{4j-1}, \alpha_{4j}$ and $\alpha_{4j-3}, \alpha_{4j-2}$. That gives 6 nonlinear equations for determination (x_j, y_j), (x_{2j-1}, y_{2j-1}), (x_{2j}, y_{2j}). Solution of the system has been computed usin modified Newton's method.

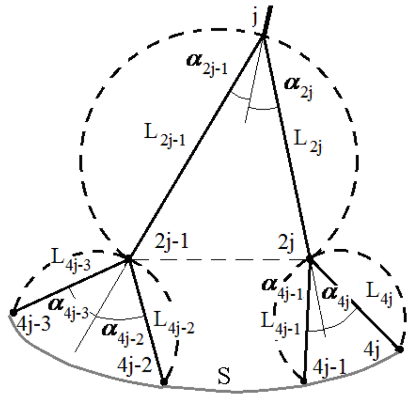


Fig.2. Algorithm of restoration the coordinates of the bifurcations from their diameters.

The trees based on the concave surface S are examples of the kidney, liver, spleen, brain vasculatures, while those based on the convex S correspond to coronary trees.

LITERATURE

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