CONDUCTIVITY OF THE CORONARY ARTERIAL TREES FOR STEADY AND WAVE BLOOD FLOWS

Filippova H

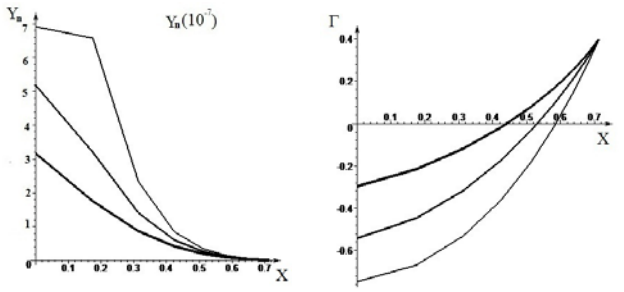
Kharkiv National Polytechnic University, Ukraine

The properties of conductivity of the human coronary arterial trees for steady and wave blood flows depend on patient specific geometry of the arterial vasculature. In [1] the results of the calculations of the steady and wave blood flow of human coronary arteries based on the measurement data carried out on plastic casts of the arteries are presented. It was shown that the admittance of the individual branching of the coronary arteries is highly variable and depends on the diameter of the feeding artery branching, while there are extensive areas with almost constant admittance, which provides the same hydrodynamic conditions for the blood supply to the appropriate regions of the heart. With adding of arterial segments, the wave admittance increases due to the reduction of the wave reflection coefficient, which after the definite number of branching becomes negative value that provides an additional blood flow to the arterial tree as it is called suction effect.

Based on the results of investigation of human coronary arteries [2], the model of the coronary vasculature as asymmetrical inhomogeneous binary tree has been built. The diameter of the input tube (the feeding artery) is assigned, while the diameters of the tubes in each bifurcation are calculated as: $d_{1j}=\dfrac{\xi_k d_{0j}}{\sqrt[\lambda_k]{1+\xi_k^{\lambda_k}}}$, $d_{2j}=\dfrac{d_{0j}}{\sqrt[\lambda_k]{1+\xi_k^{\lambda_k}}}$, where d_{0j},d_{1j},d_{2j} - the parent and the daughter tubes, ξ_k - the asymmetry coefficient, λ_k - the optimality coefficient, the value $\lambda\sim 3$, corresponds to the Murray's law. The lengths of the tubes have been calculated as $L_j=\alpha d_j^{\beta}$, α,β are constants specified for different organs [2]. Different values of λ_k and ξ_k has been assigned to the input tube, its branches and the rest of the tree according to the statistical analysis of the measurement data [3]. The tree has been generated till the diameters of the branches exceeded the minimal value $d_{min}=0.1$ mm (the capillary level). The tubes with diameters $d\le 0.1$ mm was modeled as terminal elements with admittances $Y_t=y_1+iy_2$, where y_1 and y_2 correspond to resistivity and compliance of the capillary network. The calculations of hydraulic conductivity for the steady flow have been carried out on the Poiseuille law for each tube $Y_j=\pi d_j^4/128\eta L_j$ where η is the viscosity of blood. The pressure and volumetric rate continuity conditions in each bifurcation have been assumed in the form $P_j^0\big|_{x=L_j}=P_j^1\big|_{x=0}=P_j^2\big|_{x=0}$, $Q_j^0=Q_j^1+Q_j^2$, where $x\in\left[0,L_j\right]$.

The wave blood flow has been described by linearized incompressible Navier-Stokes equations for the axisymmetric small amplitude wave propagation in thick wall tube from viscoelastic incompressible material. For the each arterial segment the characteristic admittances $Y_j^0(\omega)=\dfrac{\pi d_j^2}{4\rho_{bl}c_j(\omega)}$, where $c_i=\left(\dfrac{E_i h_i(1-F_{01})}{\rho d_i(1-\sigma_i^2)}e^{I\theta_i}\right)^{1/2}$ is the wave speed, ω is the frequency, ρ_{bl} is the density of blood, $I=\sqrt{-1}$, $P_i^0=P_i\big|_{x_i=0}$, $F_i=2J_1(\beta_i)/(\beta_i J_0(\beta_i))$, $\beta_i=\alpha_i(-1)^{3/4}$, $\alpha_i=R_i\sqrt{\omega\rho_{bl}/\eta}$ – Womersley's number, $J_{0,1}$ are Bessel functions, $h_i,E_i,\sigma_i,\theta_i$ are wall thickness, Young modulus, Poisson ratio and viscosity of the wall material. Then the input wave admittances accounting for the wave reflections at the ends of the tubes can be calculated as $Y_{in_j}=Y_j^0\dfrac{1-\Gamma_j e^{-2i\omega L_j/c_j}}{1+\Gamma_j e^{-2i\omega L_j/c_j}}$, where $\Gamma=\dfrac{Y_1^0-Y_2^0-Y_3^0}{Y_1^0+Y_2^0+Y_3^0}$ is the wave reflection coefficient at the branching. The wave admittance is used to characterize the dynamic properties of the coronary vasculature. The input admittance of the system is defined as the relationship of the flow rate Q and pressure P amplitudes at the inlet of the feeding artery.

The values of wave admittance and reflection coefficient has been computed as functions of the total length X from the inlet to the given location inside the tree $Y(X)$ and $\Gamma(X)$ at different microcirculation parameters $Re(Y_t)$ and $Im(Y_t)$. The corresponding plots $Y(X)$ and $\Gamma(X)$ are presented in Fig.1a and Fig.1b.



a b

Fig.1. Dependencies $Y(X)$ (a) and $\Gamma(X)$ (b) at different values $Re(Y_t)$ and $Im(Y_t)$

The computed results are in a good correspondence with the similar dependencies computed on the morphometric data of the coronary arteries [1]. The proposed model may be used for biomechanical interpretation of the pulse wave curves $P(t)$, $Q(t)$ for medical diagnostics.

ЛИТЕРАТУРА

1. Zenin O., Kizilova N., Filippova H. The investigation of coronary arterial tree for steady and wave blood flow. // Collected papers of XI international conference "Contemporary problems of continuum mechanics". Rostov-na-Donu. - 2007. - V.2. - P.93-97.
2. Dawson Ch.A., Krenz G.S., Karau K.L., et al. Structure-function relationships in the pulmonary arterial tree // J.Appl.Physiol. – 1999. – V.86. - P.569-583.
3. Zenin O., Kizilova N., Filippova H. Investigation of the structure of human coronary vasclature. // Biophysics. - 2007. - V.52,№5. - P.924-930.