CONDUCTING SYSTEMS OF PLANT LEAVES AS OPTIMAL NETWORKS FOR FLUID DELIVERY

[1]Szekeres A., [2]Kizilova N.

[1] Budapest University of Technology and Economics, Hungary
[2] V.N. Karazin Kharkov National University, Ukraine

Nature inspired solutions for biomimetic design of the technical and biomedical units based on the fluid/gas transportation through a network of ducts/tubes are widely discussed in recent literature [1,2]. The regularities between the lengths L, diameters D and bifurcation angles $\alpha_{1,2}$ of the tubes of different size and branching order (fig.1) can be obtained from the solution of the correspondent optimization problem for a simple cell feded by a single tube of given L and D [3].

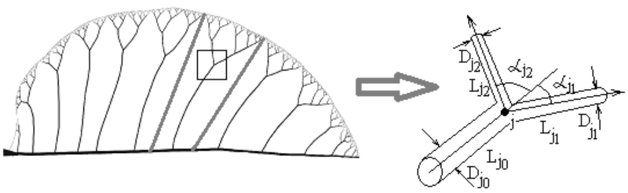

Fig. 1. A sketch of the half leaf blade and the bifurcation of tubes as design unit of the water transportation system

A concept of the microcirculatory cell as a unit of construction of the complex fluid delivery, heat and mass exchange systemswith microfluid as a working liquid substance is studied here.

Let us consider a rectangular microcirculatory cell of volume $V = V_I + V_2$, where

$$V_I = \{x \in [0,L], y \in [-a,a], z \in [-h,h]\},$$

$$V_{II} = \{x \in [0,L], y \in [-H-a,-a[\cup[a,a+H], z \in [-h,h]\},$$

$h << \{H,L\}$, (x,y,z) is the Cartesian coordinate system. The volumes V_I and V_{II} are filled with anisotropic porous media with different permeability. The model corresponds to the leaf vein (V_I) supplying its drainage area (V_{II}) by water and nutrition.

Flow is driven by the hydrostatic pressure difference at the inlet (P^+) and outlet (P^-) of the tube, as well as the osomitic pressure difference in the sap (π^+) and in the volume V_{II} (π^-). Flow in the cell is governed by Darcy law for viscous incompressible liquid. The governing equations for V_I and V_{II} are

$$\text{in } V_I: \quad \operatorname{div}\vec{U} = 0, \quad \vec{U} = -\frac{\hat{K}}{\mu}\nabla P, \qquad (1)$$

$$(\vec{U}\nabla)C = -\operatorname{div}(D_c \nabla C) \qquad (2)$$

$$\text{in } V_{II}: \quad \operatorname{div}\vec{v} = 0, \quad \vec{v} = -\frac{\hat{M}}{\mu}(\nabla p - \zeta\nabla\pi), \qquad (3)$$

$$(\vec{v}\nabla)b = -\operatorname{div}(D_b \nabla b) - q_b \qquad (4)$$

where $\vec{U}$ and $\vec{v}$, P and p, $\hat{K}$ and $\hat{M}$, C and b, D_C and D_b are the flow velocities, hydrostatic pressures, permeability tensors for the liquid, concentrations and diffusion coefficients in V_I and V_{II} accordingly; π is the osmotic pressure in V_{II}; μ is the fluid viscosity; q_b is the absorption rate of the dissolved substances in the volume (catalytic layer, live cells, etc); ζ is the selectivity coefficient governing the relative conductivity for the dissolved substances (provided they are large), so $\zeta\hat{M}$ is conductivity for the chemical components.

The boundary conditions with the Maxwell first order velocity slip model at the wall within the Knudsen layer are assumed in the form [4]

$$\text{in } V_I: \quad x = 0: \quad C = C^+, \quad P = P^+,$$

$$x = L: \quad C = C^-, P = P^-,$$

$$z = 0; \quad y = 0: \quad U_y = 0, \quad U_z = 0, \qquad (5)$$

$$y = \pm a: \; U_{x,z} = \frac{2-\sigma_{x,z}}{\sigma_{x,z}} Kn \frac{\partial U_{x,z}}{\partial U_{x,z}}, \quad U_y = \pm V_f,$$

$$z = \pm h: \quad U_z = w_\pm^I.$$

$$\text{in } V_{II}: \quad x = 0; L: \quad \frac{\partial b}{\partial x} = 0, \quad \frac{\partial p}{\partial x} = 0,$$

$$y = \pm(a+H): \quad \frac{\partial b}{\partial y} = 0, \quad \frac{\partial p}{\partial y} = 0, \qquad (6)$$

$$z = \pm h: \quad v_z = w_\pm^{II}, \quad \frac{\partial b}{\partial z} = \alpha b w_\pm^{II},$$

where $C^\pm$ are the concentrations at the inlet and outlet of the duct V_I; $w_\pm^I$ and $w_\pm^{II}$ are the mass exchange rates per unit surface area at the upper (+) and lower (-) surfaces of V_I and V_{II}; $\alpha = \text{const}$ corresponds to washing out the unabsorbed components, $\alpha = 0$ when the outer walls of the cell are impermeable for the components, n is the normal direction (coordinate) to the wall, $Kn = \lambda / L$ is the Knudsen number, λ is the mean free path of the microparticle in the fluid, L is the length, σ is the tangential momentum accommodation factor (or friction); σ =1 for purely diffuse reflection.

The lumped parameter model have been obtained from (1)-(6) by averaging over y,z. The solution has been found in analytical form and analized. The problem of hydrodynamically interacting cells is studied. Possible applications in the technical heat and mass exchangers, microheaters/coolers and fuel cells are discussed.

LITERATURE

1. Kizilova N., Szekeres A. Nature Inspired Optimal Composites. In: Continuum physics and engineering applications. – 2010. – P. 30–42.
2. Hamadiche M., Kizilova N. Nature inspired optimal design of heat conveying networks for advanced fiber-reinforced composites. // J. Thermal Eng. – 2015. – V.1, Issue 7. – P. 636-645.
3. Kizilova N. Liquid filtration in a microcirculatory cell of the plant leaf. // Intern. J. Fluid Mech. Res. – 2007. – v.34, N6. – P.572-588.
4. Karniadakis G.E., Beskok A., Aluru N. Microflows and nanoflows: Fundamentals and simulation. Interdisc. Appl. Math. Series, V.29. 2005. P.51-77.