THE PARTIAL AVERAGING OF FUZZY DIFFERENTIAL INCLUSIONS

*[1]Komleva T.A., [2]Plotnikova L.I.

[1] Odessa State Academy Civil Engineering and Architecture, Ukraine,
[2]Odessa National Polytechnic University, Ukraine.

Let $comp(R^n)(conv(R^n))$ be a family of all nonempty (convex) compact subsets from the space R^n with the Hausdorff metric

$$h(A,B)=\min_{r\ge 0}\{S_r(A)\supset B,\ S_r(B)\supset A\},$$

where $A,B\in comp(R^n)$, $S_r(A)$ is r-neighborhood of set A.

Let E^n be a family of all $u:R^n\to[0,1]$ such that u satisfies the following conditions: 1) u is normal; 2) u is fuzzy convex; 3) u is upper semicontinuous; 4) the closure of the set $\{x\in R^n : u(x)>0\}$ is compact.

If $u\in E^n$, then u is called a fuzzy number, and E^n is said to be a fuzzy number space.

Definition 1. *The set $\{x\in R^n : u(x)\ge\alpha\}$ is called the α-level $[u]^\alpha$ of a fuzzy number $u\in E^n$ for $0<\alpha\le 1$. The closure of the set $\{x\in R^n : u(x)>0\}$ is called the 0-level $[u]^0$ of a fuzzy number $u\in E^n$.*

Theorem 1. (Stacking Theorem). *If $u\in E^n$ then*

1) $[u]^\alpha\in conv(R^n)$ for all $\alpha\in[0,1]$;

2) $[u]^{\alpha_2}\subset[u]^{\alpha_1}$ for all $0\le\alpha_1\le\alpha_2\le 1$;

3) if $\{\alpha_k\}$ is a nondecreasing sequence converging to $\alpha>0$, then $[u]^\alpha=\bigcap_{k\ge 1}[u]^{\alpha_k}$.

Conversely, if $\{A_\alpha:\alpha\in[0,1]\}$ is the family of subsets of R^n satisfying conditions 1) - 3) then there exists $u\in E^n$ such that $[u]^\alpha=A_\alpha$ for $0<\alpha\le 1$ and $[u]^0=\overline{\bigcup_{0<\alpha\le 1}A_\alpha}\subset A_0$.

Let θ be the fuzzy number defined by $\theta(x)=0$ if $x\ne 0$ and $\theta(0)=1$.

Define $D:E^n\times E^n\to[0,\infty)$ by the relation

$$D(u,v)=\sup_{0\le\alpha\le 1}h\left([u]^\alpha,[v]^\alpha\right).$$

Consider the fuzzy differential inclusion

$$\dot{x}\in F(t,x),\ \ x(0)\in X_0, \qquad (1)$$

where $x\in R^n$, $t\in[0,T], F:[0,T]\times R^n\to E^n, X_0\in E^n$.

We interpret (1) as a family of differential inclusions

$$\dot{x}_\alpha\in[F(t,x_\alpha)]^\alpha,\ \ x_\alpha(0)\in[X_0]^\alpha,\ \alpha\in[0,1]. \qquad (2)$$

An $\alpha-$solution $x_\alpha(\cdot)$ of (1) is understood to be an absolutely continuous function $x_\alpha:[0,T]\to R^n$ which satisfies (2) almost everywhere. Let X_α denote the $\alpha-$solution set of (2) and $X_\alpha(t)=\{x_\alpha(t):x_\alpha(\cdot)\in X_\alpha\}$.

Clearly a family of subsets $X_t=\{X_\alpha(t):\alpha\in[0,1]\}$ can not satisfy to conditions of Theorem 1.

Therefore we will consider a R-solution of fuzzy differential inclusion (1).

Definition 2 [1]. *The upper semicontinuous fuzzy mapping $X:[0,T]\to E^n$ which satisfy to the following system*

$$\sup_{\alpha\in[0,1]}h\left([X(t+\sigma)]^\alpha,\ \bigcup_{x\in[X(t)]^\alpha}\{x+\int_t^{t+\sigma}[F(s,x)]^\alpha ds\}\right)=o(\sigma),$$
$$X(0)=X_0$$

is called the R-solution of differential inclusion (1).

Consider fuzzy differential inclusion with a small parameter

$$\dot{x}\in\varepsilon F(t,x),\ \ x(0)\in X_0, \qquad (3)$$

where $x\in R^n, t\in R_+, F:R_+\times R^n\to E^n, X_0\in E^n$, and $\varepsilon>0$ be a small parameter.

We associate with the inclusion (3) the following partial averaged fuzzy differential inclusion

$$\dot{y}\in\varepsilon G(t,y),\ \ y(0)\in X_0, \qquad (4)$$

where $G:R_+\times R^n\to E^n$ such that

$$\lim_{T\to\infty}D\left(\frac{1}{T}\int_0^T F(t,x)dt,\frac{1}{T}\int_0^T G(t,x)dt\right)=0. \qquad (5)$$

Theorem 2. *Let in domain $Q=\{(t,x):t\ge 0,$ $x\in D\in conv(R^n)\}$ the following conditions hold:*

1) mappings $F(\cdot,x)$, $G(\cdot,x)$ are measurable on R_+;

2) mappings $F(t,\cdot)$, $G(t,\cdot)$ satisfy a Lipschitz condition with a constant $\lambda>0$;

3) there exists $\gamma>0$ such that $D(F(t,x),\theta)\le\gamma$, $D(G(t,x),\theta)\le\gamma$ for almost every $t\in[0,T]$ and every $x\in R^n$;

4) for all $\beta\in[0,1], x',x''\in R^n$ and almost every $t\in[0,T]$

$$\beta F(t,x')+(1-\beta)F(t,x'')\subset F(t,\beta x'+(1-\beta)x''),$$
$$\beta G(t,x')+(1-\beta)G(t,x'')\subset G(t,\beta x'+(1-\beta)x'').$$

5) limit (5) exists uniformly with respect to x in the domain Q;

6) for any X_0 ($[X_0]^0\subset D'\subset D$), $\varepsilon\in(0,\nu]$ and $t>0$ the R-solution of the inclusion (4) together with a $\rho-$neighborhood belong to the domain G, i.e. $[X(t)]^0+S_\rho(0)\subset D$ for every $t>0$.

Then for any $\eta\in(0,\rho]$ and L > 0 there exists $\varepsilon_0(\eta,L)>0$ such that for all $\varepsilon\in(0,\varepsilon_0]$ and $t\in[0,L\varepsilon^{-1}]$ the following inequality holds

$$D(X(t),Y(t))<\eta \qquad (4)$$

where $X(\cdot),Y(\cdot)$ are the R-solutions of initial and partial averaged inclusions.

LITERATURE

1. Plotnikov A.V., Komleva T.A. The partial averaging of fuzzy differential inclusions on finite interval. // International Journal of Differential Equations. – 2014. – v. 2014, Article ID 307941, 5 pages. doi: 10.1155/2014/307941.