# FUZZY DIFFERENTIAL INCLUSION. R-SOLUTION

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Let $comp(R^n)(conv(R^n))$ be a family of all nonempty (convex) compact subsets from the space $R^n$ with the Hausdorff metric

$$h(A,B)=\min_{r\geq 0}\{S_r(A)\supset B,\ S_r(B)\supset A\},$$

where $A,B\in comp(R^n)$, $S_r(A)$ is $r$-neighborhood of set $A$.

Let $E^n$ be a family of all $u:R^n\to[0,1]$ such that $u$ satisfies the following conditions: 1) $u$ is normal; 2) $u$ is fuzzy convex; 3) $u$ is upper semicontinuous; 4) the closure of the set $\{x\in R^n : u(x)>0\}$ is compact.

If $u\in E^n$, then $u$ is called a fuzzy number, and $E^n$ is said to be a fuzzy number space.

**Definition 1.** *The set* $\{x\in R^n : u(x)\geq\alpha\}$ *is called the* $\alpha$*-level* $[u]^\alpha$ *of a fuzzy number* $u\in E^n$ *for* $0<\alpha\leq 1$. *The closure of the set* $\{x\in R^n : u(x)>0\}$ *is called the* $0$*-level* $[u]^0$ *of a fuzzy number* $u\in E^n$.

It is clearly that the set $[u]^\alpha\in conv(R^n)$ for all $0\leq\alpha\leq 1$.

**Theorem 1.** (Stacking Theorem). *If* $u\in E^n$ *then*

*1)* $[u]^\alpha\in conv(R^n)$ *for all* $\alpha\in[0,1]$;

*2)* $[u]^{\alpha_2}\subset[u]^{\alpha_1}$ *for all* $0\leq\alpha_1\leq\alpha_2\leq 1$;

*3) if* $\{\alpha_k\}$ *is a nondecreasing sequence converging to* $\alpha>0$, *then* $[u]^\alpha=\bigcap_{k\geq 1}[u]^{\alpha_k}$.

*Conversely, if* $\{A_\alpha:\alpha\in[0,1]\}$ *is the family of subsets of* $R^n$ *satisfying conditions 1) - 3) then there exists* $u\in E^n$ *such that* $[u]^\alpha=A_\alpha$ *for* $0<\alpha\leq 1$ *and* $[u]^0=\overline{\bigcup_{0<\alpha\leq 1}A_\alpha}\subset A_0$.

Let $\theta$ be the fuzzy number defined by $\theta(x)=0$ if $x\neq 0$ and $\theta(0)=1$.

Define $D:E^n\times E^n\to[0,\infty)$ by the relation

$$D(u,v)=\sup_{0\leq\alpha\leq 1}h\left([u]^\alpha,[v]^\alpha\right).$$

Then $D$ is a metric in $E^n$. Further we know that:

i) $(E^n,D)$ is a complete metric space,

ii) $D(u+w,v+w)=D(u,v)$ for all $u,v,w\in E^n$,

iii) $D(\lambda u,\lambda v)=|\lambda|D(u,v)$ for all $u,v\in E^n$ and $\lambda\in R$.

Consider the fuzzy differential inclusion

$$\dot{x}\in F(t,x),\ \ x(0)\in X_0, \tag{1}$$

where $x\in R^n$, $t\in[0,T], F:[0,T]\times R^n\to E^n, X_0\in E^n$.

We interpret (1) as a family of differential inclusions

$$\dot{x}_\alpha\in[F(t,x_\alpha)]^\alpha,\ \ x_\alpha(0)\in[X_0]^\alpha,\ \alpha\in[0,1]. \tag{2}$$

An $\alpha-$solution $x_\alpha(\cdot)$ of (1) is understood to be an absolutely continuous function $x_\alpha:[0,T]\to R^n$ which satisfies (2) almost everywhere. Let $X_\alpha$ denote the $\alpha-$solution set of (2) and $X_\alpha(t)=\{x_\alpha(t): x_\alpha(\cdot)\in X_\alpha\}$. Clearly a family of subsets $X_t=\{X_\alpha(t):\alpha\in[0,1]\}$ can not satisfy to conditions of Theorem 1.

Therefore we will consider a R-solution of fuzzy differential inclusion (1).

**Definition 2** [1]. *The upper semicontinuous fuzzy mapping* $X:[0,T]\to E^n$ *which satisfy to the following system*

$$\sup_{\alpha\in[0,1]}h\left([X(t+\sigma)]^\alpha,\ \bigcup_{x\in[X(t)]^\alpha}\{x+\int_t^{t+\sigma}[F(s,x)]^\alpha ds\}\right)=o(\sigma),$$

$$X(0)=X_0$$

*is called the R-solution* $X(\cdot)$ *of differential inclusion (1), where* $\lim_{\sigma\to 0_+}\frac{o(\sigma)}{\sigma}=0$.

**Theorem 2** [1]. *Suppose that following conditions hold:*

*1) fuzzy mapping* $F(\cdot,x)$ *is measurable, for all* $x\in R^n$;

*2) there exists* $\lambda>0$ *such that for all* $x',x''\in R^n$

$$D(F(t,x'),F(t,x''))\leq\lambda\|x'-x''\|$$

*for almost every* $t\in[0,T]$;

*3) there exists* $\gamma>0$ *such that* $D(F(t,x),\theta)\leq\gamma$ *for almost every* $t\in[0,T]$ *and every* $x\in R^n$;

*4) for all* $\beta\in[0,1],\ x',x''\in R^n$ *and almost every* $t\in[0,T]$

$$\beta F(t,x')+(1-\beta)F(t,x'')\subset F(t,\beta x'+(1-\beta)x'').$$

*Then there exists a unique R-solution* $X(\cdot)$ *of fuzzy system (1) defined on the interval* $[0,\tau]\subseteq[0,T]$.

Also we consider the differential inclusion

$$\dot{y}\in G(t,y),\ \ y(0)\in Y_0, \tag{5}$$

where $y\in R^n$, $t\in[0,T], F:[0,T]\times R^n\to E^n, Y_0\in E^n$.

**Theorem 3.** *Let* $F(t,x)$ *and* $G(t,y)$ *satisfy conditions 1)-4) of Theorem 2 and there exist* $\eta>0$ *and* $\mu>0$ *such that* $D(F(t,x),G(t,x))<\eta$ *and* $D(X_0,Y_0)<\mu$ *for every* $x\in R^n$ *and* $t\in[0,T]$.

*Then* $D(X(t),Y(t))\leq\mu e^{\lambda t}+\frac{\eta}{\lambda}(e^{\lambda t}-1)$ *for every* $t\in[0,T]$.

LITERATURE

1. Plotnikov A.V., Komleva T.A. The Partial Averaging of Fuzzy Differential Inclusions on Finite Interval. // International Journal of Differential Equations. – 2014. – v. 2014, Article ID 307941, 5 pages. doi: 10.1155/2014/307941