FUZZY INTEGRAL EQUATIONS

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Integral equations are encountered in various fields of science and in numerous applications, including elasticity, plasticity, heat and mass transfer, oscillation theory, fluid dynamics, filtration theory, electrostatics, electrodynamics, biomechanics, game theory, control, queuing theory, electrical engineering, economics, and medicine. In this report we consider fuzzy integral equation and prove the existence and uniqueness theorem, the theorem of continuous dependence on the right-hand side and initial fuzzy set and justify the possibility of using the averaging method.

Let E^n be the space of mappings $x: R^n \to [0,1]$ satisfying the following conditions: 1) x is normal; 2) x is fuzzy convex; 3) x is upper semicontinuous; 4) a closure of the set $\{y \in R^n : x(y) > 0\}$ is compact with the metric

$$D(x,y) = \sup_{\alpha\in[0,1]} h\left([x]^\alpha, [y]^\alpha\right).$$

Consider the fuzzy integral equation

$$x(t) = x_0 + \int_a^t f(t,s,x(s))ds, \qquad (1)$$

where $t \in [a,b] \subset R$ is time, $x:[a,b] \to E^n$ is a phase variable, the fuzzy mapping $f:[a,b]\times[a,b]\times E^n \to E^n$.

Definition. A continuous mapping $x:[a,b] \to E^n$ is called the solution of equation (1) on the interval $[a,b]$ if it satisfies equation (1) for all $t \in [a,b]$.

Theorem 1. Let in the domain $Q = \{(t,s,x): t,s \in [a,b], x \in E^n\}$ the fuzzy mapping $f: Q \to E^n$ be continuous and satisfy Lipschitz condition in x with constant $\lambda \ge 0$. Then integral equation (1) has a unique solution.

Theorem 2. Let in the domain Q the fuzzy mappings $f, g: Q \to E^n$ be continuous. Consider the following fuzzy integral equations (1) and

$$y(t) = y_0 + \int_a^t g(t,s,y(s))ds. \qquad (2)$$

Suppose that

1) the mapping $f: Q \to E^n$ satisfies Lipschitz condition in x with the constant $\lambda \ge 0$, denote by $x^*(\cdot)$ the unique solution of equation (1);

2) there exist nonnegative constants η_1 and η_2 such that $D(f(t,s,x), g(t,s,x)) \le \eta_1$ for all $(t,s,x) \in Q$ and $D(x_0, y_0) \le \eta_2$;

3) equation (2) has a solution $y^*(\cdot)$.

Then $D^*(x^*, y^*) =$

$$= \max_{t\in[a,b]} D(x^*(t), y^*(t))e^{-\tau(t-a)} \le \frac{\eta_2 + \eta_1(b-a)}{1-\lambda/\tau},$$

where $\tau > \lambda$.

Consider the fuzzy integral equation

$$x(t) = x_0 + \varepsilon\int_0^t f(t,s,x(s))ds, \qquad (3)$$

where $t \in R_+$ is time, $x: R_+ \to G, G \subset E^n$ is a phase variable, $\varepsilon > 0$ is a small parameter.

Along with equation (3) consider the following averaged integral equation

$$\bar{x}(t) = x_0 + \varepsilon\int_0^t \bar{f}(t,\bar{x}(s))ds, \qquad (4)$$

where

$$\bar{f}(t,x) = \lim_{T\to\infty} \frac{1}{T}\int_\tau^{\tau+T} f(t,s,x)ds. \qquad (5)$$

The following theorem that establishes the proximity of solutions of equations (3) and (4) on a finite interval holds:

Theorem 3. Let in the domain

$$Q = \{(t,s,x): t,s \in R_+, x \in G \subset E^n\}$$

the following conditions fulfill:

1) the fuzzy mapping $f(t,s,x)$ is continuous and satisfies Lipschitz condition in x with the constant λ;

2) the fuzzy mapping $\int_0^t f(t,s,x(s))ds$ is equicontinuous on R_+ uniformly with respect to fuzzy continuous mappings $x: R_+ \to G$;

3) uniformly with respect to $t, \tau \in R_+, x \in G$ there exists limit (5);

4) the solution $\bar{x}(\cdot)$ of the equation (4) with $x_0 \in G' \subset G$ is defined for $t \ge 0$ and all $\varepsilon \in (0,\sigma]$ and belongs with some $\rho-$neighborhood to the domain G.

Then for any $\eta > 0$ and $L > 0$ there exists such $\varepsilon_0(\eta, L) \in (0,\sigma]$ that for all $\varepsilon \in (0,\varepsilon_0]$ and $t \in [0, L\varepsilon^{-1}]$ the inequality holds $D(x(t), \bar{x}(t)) \le \eta$, where $x(\cdot)$ and $\bar{x}(\cdot)$ are solutions of the equations (3) and (4).