ON THE BELONGING OF CHARACTERISTIC FUNCTIONS OF PROBABILITY LAWS TO CONVERGENCE CLASS

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The function $\varphi(z)=\int_{-\infty}^{+\infty} e^{izx}dF(x)$ defined for real z is called a characteristic function of the probability law F. If φ has an analytic continuation on the disk $\mathbb{D}_R=\{z:|z|<R\}, 0<R\le+\infty,$ then we call φ an analytic in $\mathbb{D}_R$ characteristic function of the law F. Further we always assume that $\mathbb{D}_R$ is the maximal disk of the analicity of φ. It is known that φ is an analytic in $\mathbb{D}_R$ characteristic function of the law F if and only if $W_F(x)=:1-F(x)+F(-x)=O(e^{-rx})(x\to+\infty)$ for every $r\in[0,R)$. Hence it follows that $\varliminf_{x\to+\infty}\frac{1}{x}\ln\frac{1}{W_F(x)}=R$. For $0\le r<R$ we put $M(r,\varphi)=\max\{|\varphi(z)|:|z|=r\}$.

Let $\Omega(R)$ be a class of positive unbounded on $(0,R)$ function Φ such that the derivative Φ' is positive continuously differentiable and increasing to $+\infty$ on $(0,R)$. For $\Phi\in\Omega(R)$ we denote by φ the inverse function to Φ', and let $\Psi(r)=r-\Phi(r)/\Phi'(r)$ be the function associated with Φ in the sense of Newton.

As in [1], we say that φ belongs to a convergence Φ-class if $\int_{r_0}^{R}\frac{\Phi'(r)\ln M(r,\varphi)}{\Phi^2(r)}dr<+\infty.$ By $V(R)$ we denote a class of positive continuousle differentiable on $(0,+\infty)$ function $v such that $v'(x)\uparrow R$ as $x\to+\infty$. The following theorem is proved in [2].

Theorem. Let $0<R\le+\infty, \Phi\in\Omega(R), \Phi'(r)/\Phi(r)$ be a function, nondecreasing on $[r_0,R)$, $\Phi'(r)>1/(R-r)$, $\Phi'(r+1/\Phi'(r))\le H_1\Phi'(r)$, $(H_1=\text{const}>0), \Phi''(r)\Phi(r)/(\Phi'(r))^2\le H_2<+\infty$ for all $r\in[r_0,R)$ and $\int_{r_0}^{R}\frac{\Phi'(r)\ln\Phi'(r)}{\Phi^2(r)}dr<+\infty$.

Suppose that φ is an analytic in $\mathbb{D}_R$ characteristic function on probability law F such that $\varlimsup_{x\to+\infty}W_F(x)e^{xR}=+\infty.$

Then in order that φ belongs to a convergence Φ-class it is necessary and in the case, when $\ln\frac{1}{W_F(x)}=v(x)\in V(R)$, it is sufficient that

$$\int_{x_0}^{\infty}\frac{dx}{\Phi'\left(\frac{1}{x}\ln\frac{1}{W_F(x)}\right)}<+\infty .$$

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