ATTRACTORS FOR A NONLINEAR THERMOELASTICITY TRANSMISSION PROBLEM FOR SHAPE MEMORY ALLOYS.

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We consider a nonlinear one-dimensional transmission problem between two thermoelastic rods made of shape memory alloys whose free energy densities has potentials of Ginzburg-Landau form. The two rods, then in equilibrium, occupy the interval $\Omega=(0,1)$, the first rod lying in $\Omega_1=(0,l_0)$ and the second in $\Omega_2=(l_0,1)$. The system of differential equations we investigate is of the form

$$\begin{aligned}&\alpha p_t+\kappa\varphi_{xx}-f_1(\varphi)-\theta=0,\\ &\varphi_t=p_{xx}, \qquad t>0, x\in\Omega_1, \qquad (1)\\ &\gamma\theta_t-\sigma\theta_{xx}-\theta p_{xx}=0\end{aligned}$$

$$\begin{aligned}&\beta q_t+\lambda\psi_{xx}-f_1(\psi)-\tau\psi=0,\\ &\psi_t=q_{xx}, \qquad t>0, x\in\Omega_2 \qquad (2)\\ &\delta\tau_t-\mu\theta_{xx}-\tau\psi q_{xx}=0\end{aligned}$$

supplemented with boundary conditions

$$\theta_x=0,\ p=0,\ \varphi_x=0, \qquad t>0, x=0 \qquad (3)$$

on the left end,

$$\tau_x=0,\ q_x=0,\ \psi=0, \qquad t>0, x=1 \qquad (4)$$

on the right end, the transmission boundary conditions

$$\begin{aligned}&\theta=\tau, \theta_x=\tau_x, p=q,\\ &p_x=q_x, \varphi=\psi, \varphi_x=\psi_x,\end{aligned} \qquad t>0, x=l_0 \qquad (5)$$

and the corresponding initial conditions.

Here φ and ψ describe share strains of rods, p and q are the velocity potentials, θ and τ are the absolute temperatures and the constants $\alpha,\beta,\delta,\gamma,\kappa,\lambda,\mu,\sigma$ are positive.

We assume that the nonlinearities satisfy the following conditions:

(i) There exist $K_i>0,\quad i=1,2$ such that

$$F_i(v)>-K_i,\quad i=1,2 \qquad (6)$$

where $F_i(v)=\int_0^v f_i(w)dw$.

(ii) $f_i(v)\in C^2(\mathbb{R}^+)$ and there exist $L_i>0$, $i=1,2$ and $0\le r<\infty$ such that

$$|f_i(v)|\le L_i(1+|v|^r) \qquad (7)$$

for any $v\in\mathbb{R}$.

Our goal is to investigate the global existence and uniqueness of solutions to the correspondent system and the long-time behavior of solutions. Our main results consists in the fact that under conditions (6), (7) and $\alpha/\beta=\kappa/\lambda$ there exists a compact global attractor of system (1)-(5) in the space

$$H=\left\{\begin{aligned}&(p,\varphi,q,\psi,\theta,\tau)\in[H^1(\Omega_1)]^2\times[H^1(\Omega_2)]^2\times L_2(\Omega_1)\\ &\times L_2(\Omega_2): p|_{x=0}=\psi|_{x=1}=0, |p=q,\varphi=\psi|_{x=l_0}\\ &\theta,\tau\ge0, \alpha\|p_x\|^2+\beta\|q_x\|^2+\kappa\|\varphi_x\|^2+\lambda\|\psi_x\|^2+\\ &\int_{\Omega_1}(F_1(\varphi)+\gamma\theta)dx+\int_{\Omega_2}(F_2(\psi)+\delta\tau)dx\le M\end{aligned}\right\}.$$

The long-time behavior of the problems describing nonlinear thermoelasticity was investigated in a number of previous works. Qin, Liu and Song [1] proved the existence of a global attractor for the model with clamped boundary conditions for the system of the form (2). In paper [2] the long-time behavior and existence of a global attractor is investigated for the system with hinged boundary conditions. The same system with stress-free boundary conditions was considered in [3]. All these systems are damped by viscosity terms. The long-time behavior of the system without viscosity of the form (1) was investigated in [4]. The entropy of the first part of the rod (1) is linear with respect to the strain. Concerning the physical background of the model we refer to [5]. From the mathematical point of view this model is more difficult to deal with. The absence of viscosity terms leads to the loss of regularity of the phase space which causes substantial difficulties in handling the nonlinearities. Moreover, the a-priory estimates required to show the asymptotic smoothness of the dynamical system can be derived only on strong positive solutions. Therefore, we are compelled to resort to a special approximation procedure to obtain the a-priory estimates for weak solutions. As a benefit of the linearity of the entropy, the dissipation of the hole system is unexpectedly assured by the nonlinear term in the equation of heat conduction (1). To the best of our knowledge, there were no results on the existence of global attractor for the transmission problems for nonlinear thermoelasticity in shape memory alloys without viscosity, therefore, the investigation the problem is important.

REFERENCES

1. Qin Y, Liu H, Song C. Global attractor for a nonlinear thermoviscoelastic system in shape memory alloys.// Proceedings of the Royal Society of Edinburgh Section A.–2008.–v.138,N.5.–P. 1103– 1135.
2. Shang C. Global attractor for the Ginzburg-Landau thermoviscoelastic systems with hinged boundary conditions.// Journal of Mathematical Analysis and Applications. – 2008. – v.343. – P. 1–21.
3. Sprekels J, Zheng S. Maximal attractor for the system of a Landau-Ginzburg theory for structural phase transitions in shape memory alloys.// Physica D. – 1998. – v.121. – P. 252– 262.
4. Fastovska T. Global attractor for thermoelasticity in shape memory alloys without viscosity.//Math. Meth. Appl. Sci.-2015.- doi: 10.1002/mma.3815. Article first published online: 14 DEC 2015.
5. Hrusa W.J., Messaoudi S.A. On formation of singularities in one-dimensional nonlinear thermoelasticity.// Archive for Rational Mechanics and Analysis. – 1990. – v.111,N.2. – P. 135–151.