THE ELECTROSTATIC POTENTIAL OF A CHARGE IN THE SPACE BETWEEN THE SECTIONAL SPHERES

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Contemporary problems of electrostatics on classical spherical surfaces are studied. They are important in theoretical sense to test the effectiveness of new methods for solving a wide range of problems. They appear, for example, in high-power electronics for storage, transmission and reception of energy. Various methods for solving of electrostatic problems are available. There are the inversion methods and the techniques of Green's functions, as well as methods of delay differential equations. The electrostatics tasks can be considered as limiting problems of electrodynamics. A method is particularly useful for engineering calculations, if it allows obtaining analytical approximate solutions. Methods of regularization of the problems allow building effective numerically-analytical algorithms.

We consider the Dirichlet problem for the Laplace equation in the space between two concentric spheres. Here we study three independent electrostatic problems for three domains in R^3. Let us place the origin of the Cartesian and spherical coordinate systems at the center of two concentric infinitely thin perfectly conducting spheres. The radiuses of the spheres are denoted by a_1, a_2 $(a_1 < a_2)$. Each section of a sphere is formed by horizontal cuttings of the sphere on the parts. Sections are separated by non-conductive insulating infinitely thin barriers. The charge is placed on the axis of symmetry (on axis OZ) in the point with the spherical coordinates $(b,0,0)$, where $b \in (a_1, a_2)$. The potential $v^{(0)}$ of charge q is presented by the Fourier series of eigen–functions of the Sturm–Liouville problem: $v^{(0)} = \dfrac{q}{4\pi}\sum_{n=0}^{\infty} P_n(\cos\theta) b^n r^{-n-1}$, $b < r$,

where $P_n(\cos\theta)$ are Legendre polynomials. The secondary potentials u_1, u_2, u_3, u_4 are presented in the same form as $v^{(0)}$ in three domains. In the first domain $(0 \le r \le a_1)$ we search for u_1; in the second domain $(a_1 \le r \le a_2)$ for u_2, u_3; in the third domain for u_4.

We use the boundary conditions on the sectional spheres to find the unknown coefficients A_n, B_n, C_n, D_n of the Fourier series for potentials u_1, u_2, u_3, u_4 respectively. Next, we apply the method of partial domains and discrete Fourier transform. We use the orthogonality of the Legendre polynomials $P_n(\cos\theta)$ with the weight $\sin\theta$ and their completeness in space $L_2(0,\pi)$. To find the coefficients B_n, C_n of potentials u_2, u_3 we obtain the following systems of two linear algebraic equations with two unknowns:

$$\begin{cases} C_n a_1^n + B_n a_1^{-n-1} + a_1^n b^{-n-1} = F_n^{(1)}, \\ C_n a_2^n + B_n a_2^{-n-1} + b^n a_2^{-n-1} = F_n^{(2)}, \end{cases}$$

where $n = 0,1,2,3,\ldots$ The right-hand side $F_n^{(1)}$, $F_n^{(2)}$ of the systems are uniquely determined by the given potentials v_{n1}, v_{n2} $(n1 = 1,2,3,\ldots,N1;\ n2 = 1,\ 2,\ 3,\ \ldots,N2)$ on the sections of the both spheres. The determinant of each system is different from zero since $a_1 < a_2$. We note that the obtained Fourier series have different rates of convergence in the different domains. We carry out additional analysis of the series and complete the regularization problem. With this goal we also use the Poisson formula. We mark that our task is a limit variant of [1]. Our task can be generalized to other spherical-conical structures.

REFERENCES

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