# PARAMETRIC INSTABILITY OF FERROFLUID IN OSCILLATING MAGNETIC FIELDS

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The stability problem for ferrofluid layers in alternating magnetic fields is studied. Parametric excitation of sound waves in ferrofluid volumes and the instability of fluid free surface in oscillating magnetic fields are of great scientific interest due to widely use of magnetizable fluid in applications and technical devices [1]. Alternating magnetic field, applied to ferrofluid free surface, can excite as standing wave structures (parametric instability) as sharp peak shapes (Rosensweig instability) [2].

The influence of magnetic field, consisting of constant and oscillating parts, on the stability of fluid free surface was studied in [3]. The linear stability problem for viscous ferrofluid layer of finite depth is reduced to infinite system of algebraic equations:

$$A_n\xi_n = \frac{k^2(\mu-1)^2}{4\pi\rho\left((\mu^2+1)th(kh)+2\mu\right)}\left\{\left(th(kh)+\frac{1}{\mu}\right)\left(\left[\xi_{n-n_z}+\xi_{n+n_z}\right]H_{0z}m_z+\left[2\xi_n+\xi_{n-2n_z}+\xi_{n+2n_z}\right]\frac{m_z^2}{4}\right)-\left(th(kh)+\mu\right)\left(\left[\xi_{n-n_\tau}+\xi_{n+n_\tau}\right]H_{0\tau}m_\tau+\left[2\xi_n+\xi_{n-2n_\tau}+\xi_{n+2n_\tau}\right]\frac{m_\tau^2}{4}\right)\right\},$$

where

$$A_n = gk+\frac{\sigma k^3}{\rho}-\frac{\nu^2}{k\,cth(kh)-q_n cth(q_n h)}\left\{q_n\left[4k^4+(q_n^2+k^2)^2\right]cth(kh)cth(q_n h)-k\left(4q_n^2k^2+(q_n^2+k^2)^2\right)-\frac{4q_nk^2(q_n^2+k^2)}{sh(kh)sh(q_nh)}\right\}-\frac{k^2(\mu-1)^2}{4\pi\rho\left((\mu^2+1)th(kh)+2\mu\right)}\left[\left(th(kh)+\frac{1}{\mu}\right)H_{0z}^2-(th(kh)+\mu)H_{0\tau}^2\right],\quad q_n^2=k^2-\frac{s+i(\alpha+n\omega)}{\nu}.$$

Here $h$ is a thickness of fluid layer, $\rho$ is a density, $\mu$ is a permeability, $\nu$ is kinematic viscosity, $\sigma$ is surface tension, $k$ is a wavenumber, $H_\tau=\left(\vec{H}\cdot\vec{k}\right)/k$ and $H_{0z}$ are horizontal and vertical constant parts of magnetic field strength; $m_\tau, m_z$ and $n_\tau\omega, n_z\omega$ are amplitudes and frequencies of oscillating magnetic fields, $s+i\alpha$ is Floquet exponent. This system can be written in form of equation for quadratic matrix pencil:

$$\left(m_z^2C^z+m_\tau^2C^\tau+m_zB^z+m_\tau B^z+A^z+A^\tau\right)\xi=0, \quad (1)$$

where $A, B$ and C are banded matrices, which depend on the frequency ratio of vertical and horizontal magnetic fields. Marginal stability curves can be determined by solving eigenvalue problem (1), where magnetic field amplitudes are eigenvalues. This method allows to study the instability problem for ferrofluid free surface, subjected to multifrequency parametric actions, and to analyze the difference in excitation mechanisms between vertical and horizontal magnetic fields and mechanical vibrations.

The problem of acoustic wave excitation in ferrofluid volume by parametric action of magnetic field can be reduced to the equation for velocity potential perturbation [4]:

$$\ddot{\varphi}+[k^2\nu_0-\frac{d}{dt}\ln(a^2)]\dot{\varphi}+k^2[a^2-\nu_0\frac{d}{dt}\ln(a^2)]\varphi=0, \quad (2)$$

where $a$ is the velocity of sound propagation in magnetizable medium. For the case of weak magnetic field, which consists of constant and oscillating parts, $a$ takes the form:

$$a^2(t)=a_0^2+\frac{(\mu-1)^2}{4\pi\rho\mu^3}\left(H_{0\tau}^2+2H_{0\tau}m_\tau\cos\omega t+\frac{m_\tau^2}{2}(1+\cos 2\omega t)\right),$$

where $a_0$ is a sound velocity in the absence of the field. Thus, the equation (2) can be reduced either to Hill equation or to quadratic matrix pencil, similarly to the previous problem. In this case magnetic field components, which are perpendicular to the direction of wave propagation in ferrofluid volume, have no influence on the sound velocity. Moreover, the velocity of sound propagation in ferrofluid along magnetic field direction is greater than in the absence of the field. Marginal stability curves for this problem form very narrow unstable regions ("tongues"), corresponding to acoustical oscillations in ferrofluid. The dependence of unstable tongues structure on the frequency and constant part of magnetic field is studied.

Therefore, for both of these problems we sought periodic solutions, corresponding to parametric instability. The magnetic field, which consists of constant and oscillating parts, has quadratic influence on the emergence of instability and leads to the twofrequency parametric action.

For the stability problem of ferrofluid free surface, unstable tongues have a diverse structure, depending on the system parameters and magnetic field orientation. In the pure oscillating magnetic field only harmonic instability tongues are the most dangerous. Appending of the constant part to oscillating field leads to appearance of subharmonic instability tongues, which with increasing of constant field become more dangerous. Consequently, the addition of constant part to alternating magnetic field may cause the transition from harmonic to subharmonic oscillations and the occurence of closed bounded instability tongues.

## LITERATURE

1. Taketomi S., Tikadzumi S. Magnetic fluids. – M.:Mir. – 1993. – 272 p. (in Russian)
2. Blums E., Cebers A., Maiorov M. M. Magnetic Fluids. – Berlin: Walter de Gruyter. – 1997. – 416 p.
3. Patsegon N. F., Potseluiev S. I. The stability of a free surface of viscous magnetizable fluid at multiparametric excitation. //Applied Hydromechanics. – 2014. – 16(88), No 3. – p. 36 – 51. (in Russian)
4. Patsegon N. F., Potseluiev S. I. The volumetric parametric resonance in magnetizable medium. //Visnyk of V. N. Karazin Kharkiv National University, Ser. Mathematics, Applied Mathematics and Mechanics. – 2015. – v.81. – p. 20 – 35.