DIFFERENT DEFINITIONS OF THE SPECTRUM ALMOST PERIODIC FUNCTIONS

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Let’s consider the various definitions of spectrum for almost periodic functions in a finite dimensional spaces for uniform, Stepanov's, Weil's, Besicovitch's metrics. It has been proved that in these cases the classical definition of spectrum is equivalent some analogues of Beurling's one.

Definition 1 (see [1]). Function f(x): $R^n \to C$ is called D-almost periodic (a.p.) function if there exists a sequence of finite exponential sums

$P_n(x) = \sum_j c_j e^{i<\lambda_j, x>}$, $c_j \in C, \lambda_j \in R^n$, for which

$$\lim_{n\to\infty} D[f(x), P_n(x)] = 0.$$

We consider D_U-umiform metric, $D_{S_l^p}$ – Stepanov’s metric, D_{W^p} – Weil’s metric and D_{B^p} – Besicovitch’s metric.

Definition 2. The spectrum of function f(x) is the set

$$spf=\{\lambda \in R^n : a(\lambda,f) \neq 0\}$$

where $a(\lambda,f)= M\{f(x)\, e^{-i<\lambda, x>}\}$.Definition 3. Beurling type spectrum of an a.p. function is $sp_B f=\{\lambda \in R^n : e^{-i<\lambda, t>} \in \overline{\mathrm{Lin}\{f(x+t)\}_{x\in R^n}}\}$.

Note that the closure is taken in the same metric D, in which we define the periodicity.

Theorem. Let f(x): $R^n \to C$ be an D-a.p. function integrable in the Riemann sense over each squared beam. Then sp f = sp_B f.

REFERENCES

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