LEVY'S PHENOMENON FOR ANALYTIC FUNCTIONS IN $\mathbb{D}\times\mathbb{C}$

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We consider

$$f(z) = f(z_1, z_2) = \sum_{n+m=0}^{+\infty} a_{nm} z_1^n z_2^m$$

with the domain of convergence $\mathbb{T} = \{z \in \mathbb{C}^2 : \ |z_1| < 1, |z_2| < +\infty\}$. By $\mathcal{A}^2$ we denote the class of analytic functions of form (2) with the domain of convergence $\mathbb{T}$ and $\frac{\partial}{\partial z_2} f(z_1, z_2) \not\equiv 0$ in $\mathbb{T}$.

For $r = (r_1, r_2) \in T := [0,1) \times [0, +\infty)$ and function $f \in \mathcal{A}^2$ we denote

$$\Delta_r = \{(t_1, t_2) \in T : t_1 > r_1, t_2 > r_2\},$$

$$M_f(r) = \max\{|f(z)| : \ |z_1| \le r_1, |z_2| \le r_2\},$$

$$\mu_f(r) = \max\{|a_{nm}| r_1^n r_2^m : \ (n,m) \in \mathbb{Z}_+^2\}.$$

We say that $E \subset T$ is set of *asymptotically finite logarithmic measure on* T if there exists $r_0 \in T$ such that

$$\nu_{\ln}(E \cap \Delta_{r_0}) := \iint_{E \cap \Delta_{r_0}} \frac{dr_1 dr_2}{(1-r_1) r_2} < +\infty, (E \in \Upsilon).$$

In [1] one can find following statement about Wiman's type inequality for analytic functions from the class $\mathcal{A}^2$: for every $\delta > 0$ there exists a set $E = E(\delta, f) \subset \Upsilon$ such that for $r \in T \setminus E$ we obtain

$$M_f(r) \le \frac{\mu_f(r)}{(1-r)^{1+\delta}} \ln^{1+\delta} \frac{\mu_f(r)}{1-r} \ln^{1/2+\delta} r_2. \qquad (1)$$

Also in [1] was proved that inequality (1) is sharp. In particular for some $f(z_1, z_2) \in \mathcal{A}^2$ we have

$$E = \{r \in T : M_f(r) > \frac{\mu_f(r)}{(1-r)} \ln \frac{\mu_f(r)}{1-r}\} \not\in \Upsilon.$$

We prove the sharp Wiman's inequality for random analytic functions in $\mathbb{T}$. We will prove, that almost surely the exponent $1+\delta$ in inequality (1) one can replace by $\frac{1}{2} + \delta$, and this exponent cannot be placed by a number smaller than $\frac{1}{2}$.

Let $Z = (Z_{nm}(t))$ be a sequence of random complex variables. $Z_{nm}(t) = X_{nm}(t) + iY_{nm}(t)$ such that both $X = X_{nm}(t)$ and $Y = Y_{nm}(t)$ are real MS ([1]). By $K(f, z)$ we denote the class of random analytic functions of the form $f(z,t) = \sum_{n+m=0}^{+\infty} a_{nm} Z_{nm}(t) z_1^n z_2^m$.

Theorem 1. *Let* $f \in \mathcal{A}^2$, Z *be a MS uniformly bounded by the number 1,* $\delta > 0$. *Then almost surely in* $K(f, z)$ *there exists a set* $E = E(f, t, \delta), E \subset \Upsilon$ *such that for all* $r \in T \setminus E$ *we have*

$$M_f(r,t) \le \frac{\mu_f(r)}{(1-r_1)^{1/2+\delta}} \ln^{1/2+\delta} \frac{\mu_f(r)}{1-r_1} \cdot \ln^{1/4+\delta} r_2. \ (2)$$

Also no one of powers $1/2+\delta$ in inequality (3) one cannot replace by smaller number than $1/2$.

Theorem 2. *Let* Z *be a sequence of random variables such that* $|Z_{nm}| \ge 1$ *for almost all* $t \in [0;1]$. *Then there exist an analytic function* $f \in \mathcal{A}^2$, *a constant* $C > 0$ *and a set* $E = E(f, t, \delta) \subset \mathbb{T}, E \not\subset \Upsilon$ *such that almost surely in* $K(f, z)$ *we get for all* $r \in E$

$$M_f(r,t) \ge \frac{C\mu_f(r)}{\sqrt{1-r_1}} \cdot \ln^{1/2} \frac{\mu_f(r)}{1-r_1}. \qquad (3)$$

LITERATURE

1. Kuryliak A.O., Shapovalovska L.O., Skaskiv O.B. Wiman's type inequality for some doble power series // Mat. Stud. – 2013. – v.39, N2. – P. 134–141.