ENTIRE DIRICHLET SERIES WITH MONOTONOUS COEFFICIENTS AND LOGARITHMIC h-MEASURE

[1] *Salo T.M.*, [2] *Skaskiv O.B.*, [2] *Panchuk S.I.*

[1] National University "Lvivs'ka Politekhnika", Lviv, Ukraine
[2] Ivan Franko National University of L'viv, Ukraine

Let L be the class of positive continuous increasing to $+\infty$ on $[0;+\infty)$ functions. By φ we denote inverse function to a function $\Phi \in \mathrm{L}$.

Let D be a class of entire (absolutely convergent in the whole complex plane C) Dirichlet series of the form $F(z)=\sum_{n=0}^{+\infty} a_n e^{z\lambda_n},\, z\in \mathrm{C},$ where a sequence (λ_n) such that $\lambda_n \in \mathbb{R}\ (n\geq 0)$, $\lambda_n \neq \lambda_k$ for any $n\neq k$ and

$$(\forall n\geq 0):\quad 0\leq \lambda_n < \beta := \sup\{\lambda_j : j\geq 0\}\leq +\infty.$$

For $F\in \mathrm{D}$ and $x\in\mathbb{R}$ we denote

$$M(x,F)=\sup\{|F(x+iy)|: y\in\mathbb{R}\},$$
$$m(x,F)=\inf\{|F(x+iy)|: y\in\mathbb{R}\},$$
$$\mu(x,F)=\max\{|a_n|e^{x\lambda_n} : n\geq 0\}.$$

D_a denotes subclass of Dirichlet series $F\in\mathrm{D}$ with a fixed sequence $a=(|a_n|)$, $|a_n|\searrow 0$ $(n_0\leq n\to+\infty)$, and $\mathrm{D}_a(\Phi)$ denotes subclass of functions $F\in \mathrm{D}_a$ such that $\Phi\in\mathrm{L}$ and $\ln\mu(x,F)\geq x\Phi(x)$ $(x\geq x_0)$. Let $\mu_n := -\ln|a_n|\,(n\geq 0).$

In the paper [1] one can find such *statement*: *For every entire function* $F\in\mathrm{D}_a$ *the relations* $M(x,F):\mu(x,F), M(x,F):m(x,F)$ *are hold as* $x\to+\infty$ *outside some set* E *of finite logarithmic measure, i.e.* $\log-\mathrm{meas}(E):=\int_E d\ln x<+\infty$, *uniformly in* $y\in\mathrm{R}$, *if and only if* $\sum_{n=n_0}^{+\infty}(\mu_{n+1}-\mu_n)^{-1}<+\infty.$ The *finiteness* of logarithmic measure of an exceptional set E in this theorem is the sharp estimate. It is follows from the next *assertion* ([2]): *For every increasing sequence* (μ_n), *such that* $\sum_{n=n_0}^{+\infty}(\mu_{n+1}-\mu_n)^{-1}<+\infty$ *and for any function* $h\in\mathrm{L}$ *there exist an entire Dirichlet series* $F\in\mathrm{D}_a$ *with* $|a_n|=\exp\{-\mu_n\}$, *a set* E *and a constant* $d>0$ *such that* $(\forall x\in E)$: $M(x,F)\geq(1+d)\mu(x,F)$, $M(x,F)\geq(1+d)m(x,F)$ *and*

$$\mathrm{h}-\log-\mathrm{meas}(E):=\int_{E\cap[1,+\infty)}h(x)d\ln x=+\infty.$$

Due to this assertion the natural **question** arises: *what conditions must satisfy the entire Dirichlet series* $F\in\mathrm{D}_a$ *that relations* $M(x,F)\sim\mu(x,F),$ $M(x,F)\sim m(x,F)$ *are hold as* $x\to+\infty$ *outside some set* E *of finite logarithmic h-measure, i.e.* $\mathrm{h}-\log-\mathrm{meas}(E)<+\infty$ *?* We give the answer to this question. Our main result is the following.

Theorem ([3]). *Let* (μ_n) *be a sequence such that* $\sum_{n=n_0}^{+\infty}(\mu_{n+1}-\mu_n)^{-1}<+\infty$, $h\in\mathrm{L}_+,\Phi\in\mathrm{L}$ *and* $F\in\mathrm{D}_a(\Phi)$. *If*

$$(\forall b>0):\quad \sum_{n=n_0}^{+\infty}h\left(\varphi(\lambda_n)\cdot\left(1+\frac{b}{\mu_{n+1}-\mu_n}\right)\right)\frac{1}{\mu_{n+1}-\mu_n}<+\infty,$$

then the relations $M(x,F)\sim\mu(x,F),$ $M(x,F)\sim m(x,F)$ are hold as $x\to+\infty$ outside some set E of finite logarithmic h-measure.

LITERATURE

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