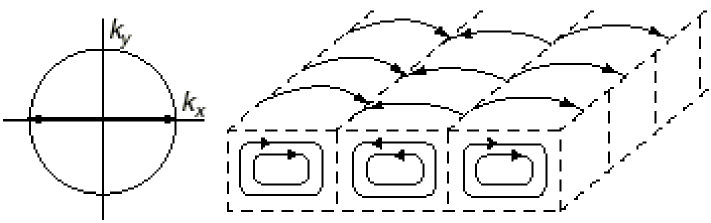
MODELING OF NOISE INFLUENCE ON THE FORMATION OF SPATIAL STRUCTURES IN THE PROCTOR–SIVASHINSKY MODEL

Gushchin I.V., Belkin E.V.

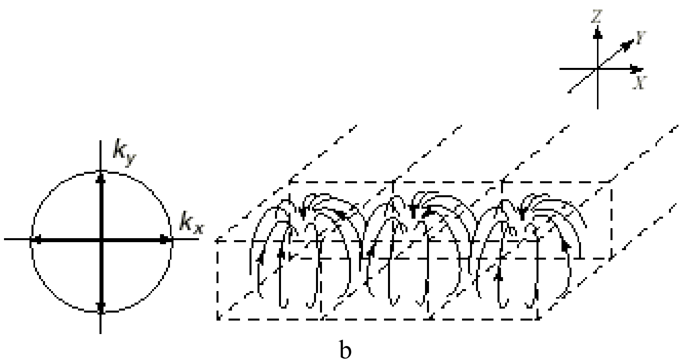
V.N. Karazin Kharkov National University, Ukraine

There are a number of stationary solutions with a small number of spatial perturbations in the problem of studying the process of convection in a thin layer of liquid. One of which is stable (convective cells) and the second is an unstable (see [1]).The study of the model [2], using a multimode description of the emergence revealed a long-lived quasi-stable state, merging into a stable state.

In a subsequent paper [3] it was shown that these structural transitions are phase transitions of the second kind. The presence of such phenomena is of interest not only for the study of convection, but also useful for understanding the dynamics of the microstructure of the phase transitions, which are described more based on phenomenology. Generally speaking, in this model, the width of the interval of instability in k – space is a ring – the average radius is equal to unity and the width of the order of the relative above-threshold. In the process of instability in the spatial spectrum develops blow-up regime, forming a discrete features – narrow spectral lines. The last stage of this process, ensuring the implementation of the spatial structure, and is the subject of this paper.



a



b

Fig.1. Convective structures: shafts (a) and square cells (b).

If the Rayleigh number Ra exceeds a critical value Ra_{thr} , i.e. $Ra = Ra_{thr}(1+\varepsilon)$, the fluid between poorly conducting horizontal surfaces heat (along axis z) there is a three-dimensional convection. (see [3]). This convection described by Proctor – Sivashinsky [1,2], which defines the dynamics of this process the temperature field in the horizontal plane (x, y):

$$\dot{\Phi} = \varepsilon^2\Phi - (1-\nabla^2)^2\Phi + \frac{1}{3}\nabla\left(\nabla\Phi\left|\Phi\right|^2\right) + \varepsilon^2 f\,, \tag{1}$$

where *f* is external additive noise; ε is sufficiently small and positive definite quantity, that threshold determines the convection progress. Under these conditions, the solution can be sought in the form of

$$\Phi = \varepsilon\sum_j a_j \exp(i\vec{k}_j\vec{r}) \tag{2}$$

where $|\vec{k}_j| = 1$. Renormalize time unit $\propto \varepsilon^2$, for slow amplitudes a_j we obtain evolutionary equation (3):

$$\dot{a}_j = a_j - \sum_{i=1}^{N} V_{ij}\,|a_i|^2\,a_j + f \tag{3}$$

Interaction coefficients are defined in (4):

$$V_{ij} = (2/3)\left(1 - 2\left(\vec{k}_i\vec{k}_j\right)^2\right) = (2/3)\left(1 + 2\cos^2\vartheta\right), \tag{4}$$

where ϑ is the angle between vectors $\vec{k}_i$ and $\vec{k}_j$, and $V_{jj} = 1$.

The picture of the temperature field at each point of the field in the interval $-Lx/2 < x < Lx/2$, $-Ly/2 < y < Ly/2$ determined by

$\Phi = \sum_{n=-N}^{n=+N} a_{n,m}\mathrm{Sin}(2\pi nx)\mathrm{Sin}(2\pi my)$, where $n = N\mathrm{Cos}\,\vartheta_s$, $m = N\mathrm{Sin}\,\vartheta_s$) are integers, and $N^2 = n^2 + m^2$.

From the initial fluctuations quickly instituted a wide range of ϑ. The value of the quadratic form $I = \sum_j a_j^2$ of this spectrum can be estimated by equating the right-hand side of (3) to zero, thus we obtain a value close to 0.75.

In the case of a large number of modes with high accuracy calculations of the system is delayed in its development, remains in dynamic equilibrium [2, 4]. For further development – "crystallization", one of the modes should receive a portion of energy greater than some threshold. That is, under these conditions requires a certain level of noise – fluctuations. This is achieved at a finite value of noise $f \neq 0$, or decreases the accuracy of calculations, which, as noted in [4] is equivalent. Such cases, when the noise can trigger or accelerate the process of instability are collected in the book [5].

If one of the modes we obtain the desired amount of energy, developing a simple process of formation of convective structures – shafts. The value of I at this reaches values close to one ($I \to 1$). However, this condition is not stable and there is a structural one: the modulation of convective rolls have along the axis of rotation of the fluid, the characteristic size is reduced. In this transition state the system is sufficiently long (which slightly increases within certain limits, by increasing the number of modes), and the value is retained $I \approx 1.07$. After a sufficiently long time, ten times larger than the inverse growth rate of the initial linear instability of the newly formed "side" spectrum "survive" only one mode (fig.3), whose amplitude is compared with the initial amplitude of the leading fashion. In the end, forming a stable convective structure – square cells in which the quadratic form of the system reaches $I = 1.2$ (see fig.2a).

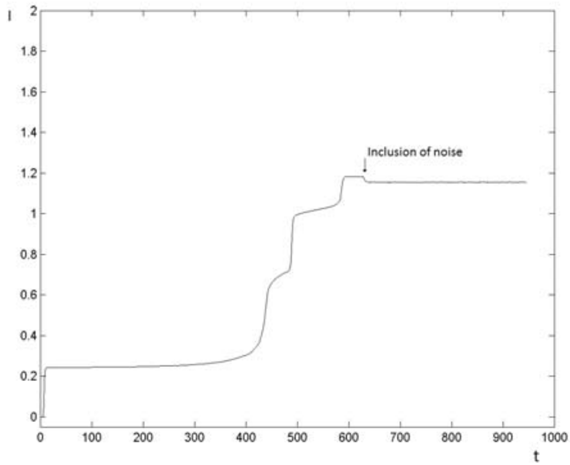


Fig.2. Changes of the quadratic form $I = \sum_j a_j^2$

a) in the absence of noise b) in the presence of noise

It was after the first spike is formed metastable structure – a system of convective rolls, and up to the second burst quadratic form $I \approx 1$ does not change. The next burst signals the appearance of metastable secondary structure with a new value $I \approx 1.07$. After the second burst of the derivative of the quadratic form begins to form a stable structure of convective cells. This behavior proves the existence of structural phase transitions in the system.

The spatial structure can have defects, whose number may vary with the introduction of noise. The spectral picture of the ideal structure of square cells having the form of two equal–amplitude modes, which are separated from each other by π/2, after the introduction of noise into the system after a time returned to the initial state of oscillation. Estimate of the number of defects allowed us to determine a direct correlation between the "spectral" and "visual" defect, quantitatively expressed by the ratio of defective cells to their total number. Calculations with great precision allow us to estimate the number of defects, depending on the level of noise. Such a defect structure can be produced using less precise in the absence of noise.

Growth of the zero amplitudes will occur when add to the established system noise after the formation of convective cells. Thus will be formation of defective structures, and increasing of noise will change convective cells to the initial amorphous state. Fig. 2b shows that the quadratic form is reduced and varies at the level I = 1.1.

Denote the mode amplitudes, which form in each realization of the spatial structure of square convective cells, as a_1 and a_2. In the interval between the second and third burst of the quadratic form we study the dynamics of "spectral defect" structure

$$D = \sum_{j\neq 1,2} a_j^2 / \sum_j a_j^2 , \qquad (5)$$

based on the squares of the amplitudes of the modes of the spectrum, which does not meet the system of square cells to the total sum of squares mod. “Visual defectiveness $d = N_{def} / N$, where N_{def} is the number of defective spatial cells (area of the structure occupied by irregular cells) and N is number of cells in a perfect regular structure (total area of the structure). Fig. 4 shows that spectral and visual defectiveness has a similar behavior. Number of defects decreases as a result of structural changes and ultimately obtained an ideal structure.

When external noise is added the growth of defect structures observed. Significance of defects increases to certain value and then does not change until the noise value will not change. This suggests that the importance of defect depends on the noise.

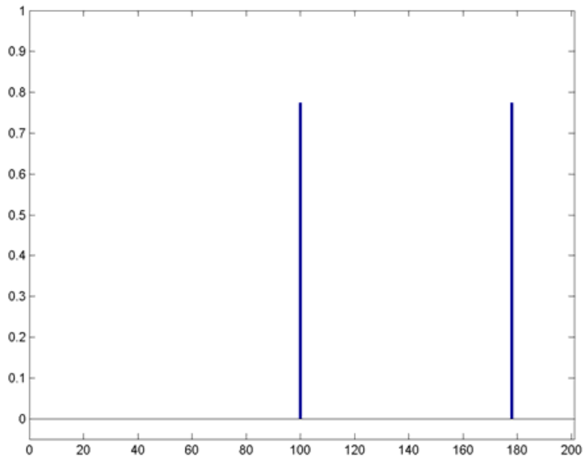


Fig.3. Amplitude spectrum. Stable convective structure

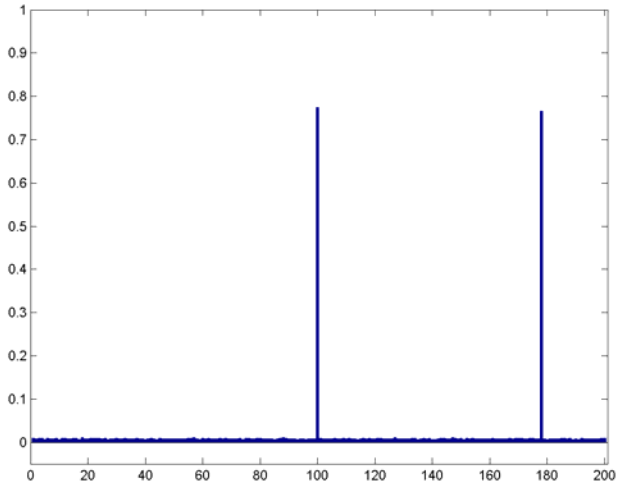


Fig.4. Amplitude spectrum. Noise influence and formation defective structure. Value of noise 0.01

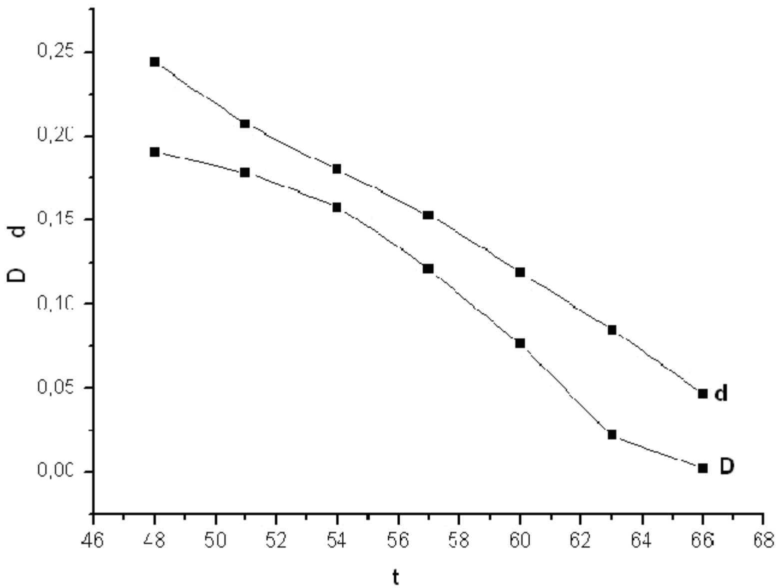


Fig.5. Spectral and visual defectiveness

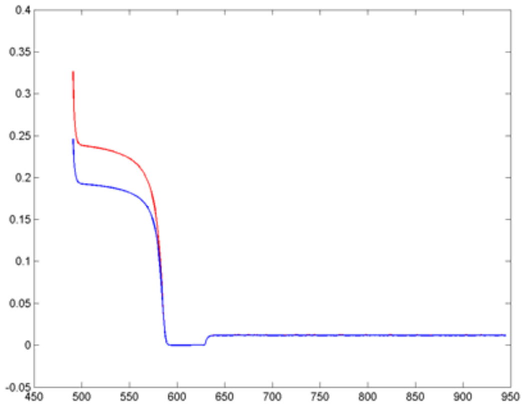


Fig.6. Noise influence on the formation of defect structures defectiveness

Conclusions. Feature of this model describing convection is the presence of the three states. Times of structural transitions between metastable states is less than the time of their existence. A qualitative and quantitative agreement between the spectral and visual defect was found. There is the influence of noise on the formation of spatial structures, including the formation of defects. It is shown that the accuracy of the calculations affect the defect structure is similar to noise. We find quantitative agreement noise (high accuracy) and precision used (in the absence of noise). The authors thank A. Kirichok and V.M. Kuklin for his attention to this work.

REFERENCES

1. Malomed B.A., Nepomnjashchij A.A., Trybelsky M.P. Twodimensional quasi periodic structures in unsteady systems. // J. Experim. Theor. Phys. – 1989. – v.96. – P.684–699. (in Russian)
2. Kirichok A.V. and Kuklin V.M. Allocated Imperfections of Developed Convective Structures. // Phys. Chem. of the Earth. Part A. – 1999. – N 6. – P.533–538.
3. Belkin E.V., Gushchin I.V., Kirichok A.V., Kuklin V.M. Structural transitions in Proctor–Sivashinsky model // VANT. Ser. Plasma electronics and new methods of acceleration. – 2010. – N4. – P.296–298. (in Russian)
4. Belkin E.V., Gushchin I.V. Mathematical model of convection of a liquid layer with temperature gradient. // KMNT. – 2010. – v.1. – P.39–40. (in Russian)
5. Horsthemke W., Lefever R. Noise–Induced Transitions: Theory and Applications in Physics, Chemistry, and Biology. (Springer Series in Synergetics). Springer. – 2007.