STUDY OF AFTER BUCKLING DEFORMATIONS ON THE EXAMPLE OF COMPOSITE RODS STRUCTURES

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Study of any mechanical system always begins from choosing a corresponding physical model or, in other words, a computation scheme. Going from real scheme to its physical model we are commonly simplify system. In that way we disengage from its particular characteristics and ignoring the many factors what in that concrete task seems to be insignificant. Thus in mechanics of deformable solids in macro physical bodies studying characteristics diverted from the molecular structure of matter and suggested that matter from body is consists are continuously fills some part of space.

In our time widespread mechanics of solids task solving methods based on discretization of partial differential equations. In same time is known what these equations are inexact and are based on averaging the actual discrete structures. With this in mind attention should be given to developing of new models that would have been deprived of those flaws what are characterized to existing models. And these new models must be computable from the beginning.

A.D. Shamrovsky, A.I. Bezverkhy and V.V. Krivulyak in the work [1] have described such method of computing based on successive displacements. It is numerical method and can be used effectively only on a computer. It is based on the idea that we have a set of nodes connected with each other by some relations. In nodes could be some effects that affect all nodes system by relations. In simple case of rod systems it is knots, rods, powers and initial displacements. Powers in nodes have some directions in which we displace them. The displacement of node is proportional to the power in it and is equal to $\mathbf{P}\gamma$ where $\mathbf{P}$ is power vector and γ some very small positive value (i.e. $0 < \gamma << |\mathbf{P}|$). When we have made all displacements then we could compute reactions what appears in all nodes connections by some law what connects displacements and powers.

These reactions give us new powers what acts to nodes and affects displacements. We continue these steps until displacements and sum of powers would not be equal to zero (some small value of preset accuracy). This method shows its great accuracy due to lack of computational errors and applicable for calculation of variety linear two-dimensional constructions problems with elastic links. But detailed study shows what it could be used to solve not only linear but geometry–nonlinear problems both in two-dimensional [2] and multidimensional cases [3] due to its simplicity. It allows solving a variety of tasks what could be modeled by set of nodes connected by elastic links.

Application of this method could be demonstrated on the simple rod constriction (Fig. 1a) computing were rods represents elastic links what connects nodes (knots). Now and in all next examples is solving problem of swivel connected rod constrictions loading. Material of rod is steel with Young's modulus equals to 2,0e+11 Pa and cross-section equals to 1,0e–4 m. The specified accuracy is 1,0e–10 and γ = 1,0e–8. The geometry of construction is showed. All results are given for illustrative purposes.

At the end of algorithm working we get new positions of all knots (Fig. 1b) and reactions in all rods.

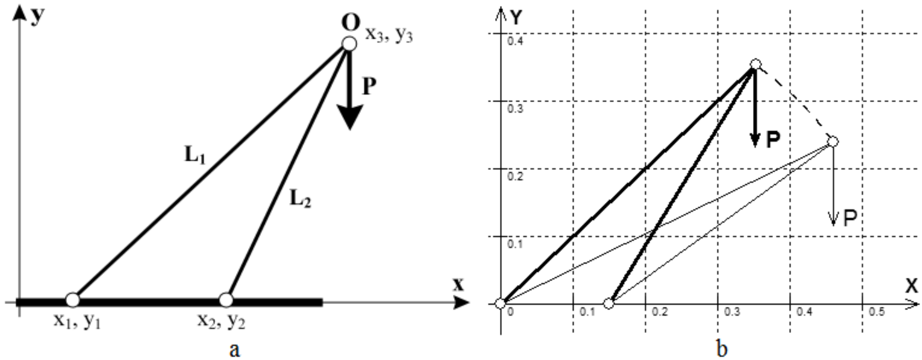

Fig. 1. Simple rod construction a) general view; b) after deformation.

In first compute for node **O** the total projection of loads on axes. Include external force **P** and rods reactions R_1, R_2:

$$\sum X = -R_1 \cos\alpha_{1x} - R_2 \cos\alpha_{2x} ,$$

$$\sum Y = -R_1 \cos\alpha_{1y} - R_2 \cos\alpha_{2y} - P .$$

In general these projections are different from zero. In particular the first approximation of reactions in rods are equal to zero and accounted only external force **P**:

$$\sum X = 0 , \ \sum Y = -P .$$

When setting displacement projections of node **O** proportionally to total loads:

$$\Delta x_3 = \gamma \sum X , \ \Delta y_3 = \gamma \sum Y .$$

In result node **O** moved in position with coordinates:

$$x_{n3} = x_3 + \Delta x_3 , \ y_{n3} = y_3 + \Delta y_3 .$$

When **O** is moved then lengths of rods are changed on values:

$$\Delta L_1 = L_{n1} - L_1 , \ \Delta L_2 = L_{n2} - L_2 ,$$

where $L_1 = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$, $L_{n1} = \sqrt{(x_{n3} - x_1)^2 + (y_{n3} - y_1)^2}$, $L_2 = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$, $L_{n2} = \sqrt{(x_{n3} - x_2)^2 + (y_{n3} - y_2)^2}$ — initial and final lengths of first and second rods.

After these deformations in rods appear some reactions:

$$R_1 = C_1 \Delta L_1 , \ R_2 = C_2 \Delta L_2 ,$$

where C_1, C_2 — stiffness of first and second rods.

These reactions are directed along new rods lines what they receive in result of node **O** displacement.

Now total projections of loads what affect on node **O** will be:

$$\sum X = -R_1 \cos\alpha_{n1x} - R_2 \cos\alpha_{n2x} ,$$

$$\sum Y = -P - R_1 \cos\alpha_{n1y} - R_2 \cos\alpha_{n2y} ,$$

where $\cos\alpha_{n1x} = \frac{x_{n3} - x_1}{L_{n1}}$, $\cos\alpha_{n1y} = \frac{y_{n3} - y_1}{L_{n1}}$, $\cos\alpha_{n2x} = \frac{x_{n3} - x_2}{L_{n2}}$, $\cos\alpha_{n2y} = \frac{y_{n3} - y_2}{L_{n2}}$.

If computed sums are not equal to zero (much more different from zero), then procedure is repeated starting from second step with change: $x_3 = x_{н3}$, $y_3 = y_{н3}$.

Recurrent process is ends with achieving of equilibrium state of **O** node with specified accuracy.

In more complex constructions (Fig. 2) the finding of load projections is same as previous.

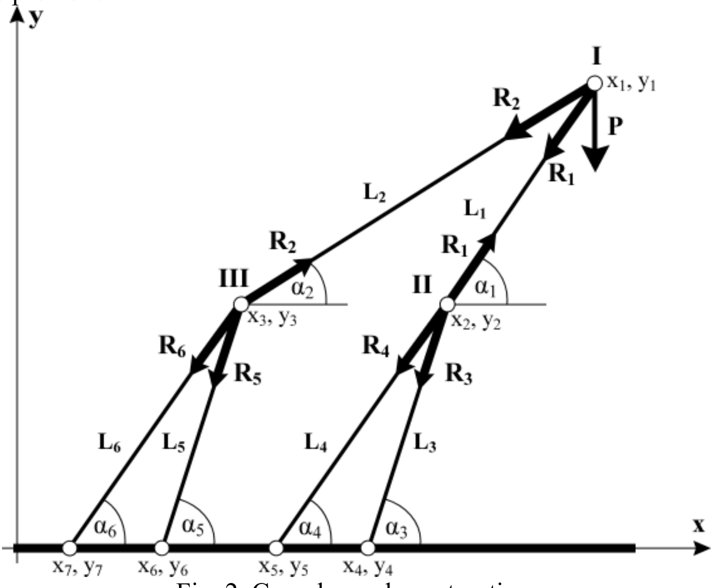

Fig. 2. Complex rod construction.

The only difference is what the external force dose not directly affects displacements of nodes II and III:

$$\sum X_I = -R_1 \cos\alpha_{1x} - R_2 \cos\alpha_{2x} ,$$

$$\sum Y_I = -R_1 \cos\alpha_{1y} - R_2 \cos\alpha_{2y} - P\ ,$$
$$\sum X_{II} = R_1 \cos\alpha_{1x} - R_3 \cos\alpha_{3x} - R_4 \cos\alpha_{4x}\ ,$$
$$\sum Y_{II} = R_1 \cos\alpha_{1y} - R_3 \cos\alpha_{3y} - R_4 \cos\alpha_{4y}\ ,$$
$$\sum X_{III} = R_2 \cos\alpha_{2x} - R_5 \cos\alpha_{5x} - R_6 \cos\alpha_{6x}\ ,$$
$$\sum Y_{III} = R_2 \cos\alpha_{2y} - R_5 \cos\alpha_{5y} - R_6 \cos\alpha_{6y}\ .$$

The general algorithm of computing that is used by this method is described below:

An arbitrary geometric structures consisting of rods and connected pivotally at the nodes is considered. Nodes can be loaded by concentrated forces.

Preliminary efforts in all the rods and moving of all nodes are considered equal by zero.

First step: Calculate the sum of the projections on the axes of all forces that acting to each node, accounting both loads and tensions in the rods.

Second step: For each node given the increments of projections of displacements that are proportional to the calculated effort projections on the first step. These increments are added to those found earlier movements of nodes. Thus, there is a new position of the rod.

Third step: Based on the new position of the rod the new length is calculated. And according deformation with proportional force and guide cosines are received too. Now return to the first step.

Recurrent process ends then the equilibrium state is reached, its mean the sufficiently small absolute values of the nods displacements. Thus the required accuracy of calculation can be specified. However increased accuracy leads to an increase of time required to achieve it.

Integration of non-linearity in computing of these structures gives ability to see buckling its mean when with small load incising significant raise of deformation happens (Fig. 3).

Features of this method make it easy to discover this phenomenon and find critical load leads to buckling. But in difference of existing methods what intended for finding these loads, what are limited in finding only first buckling load (Fig. 4), this method gives ability to continue loading after buckling and find other buckling and critical loads what leads to them (Fig. 5).

Another example of double buckling can be seen on more complex construction (Fig. 6).

Then load increases the first buckling occurs and if it’s continue to incise the second buckling occurs (Fig. 7).

As it was abovementioned, this method allows solving not only two-dimensional but multidimensional problems. Simple example of three-dimensional buckling (Fig. 8) shows result (Fig. 8b) equivalent to two-dimensional case (Fig. 3b).

More interesting result could be showed constructions with elements what have different stiffness (Fig. 9).

They could lose stability more then twice (Fig. 10). It's depend of hove many supports construction have.

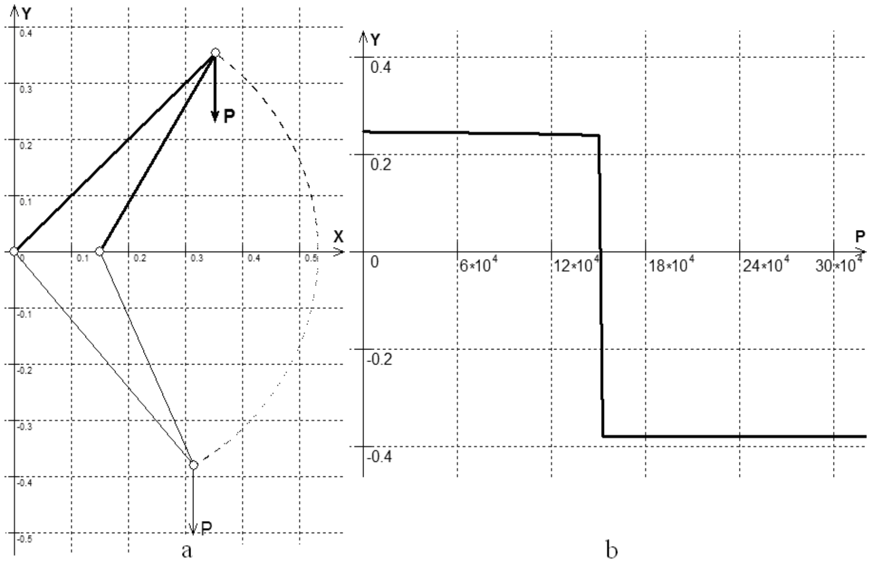

Fig. 3. Simple rod construction after buckling: a) general view; b) dependence of the displacement from the applied force.

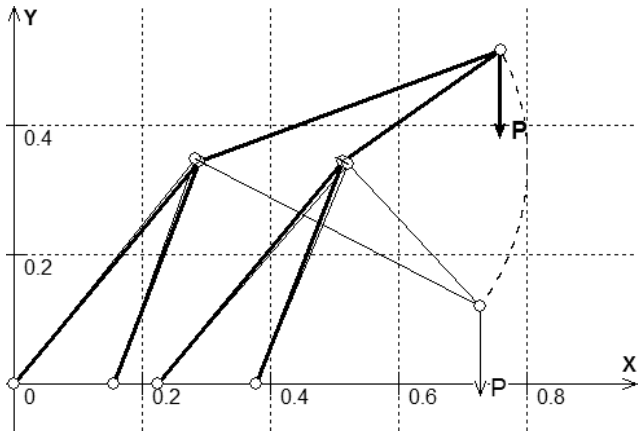

Fig. 4. First buckling in complex rod construction.

For observed construction the base load sets in researched node and then the calculation begins. After its end the new position of observed node marked. Then load increases on small amount and calculation begins again. When the difference from previous position are much more bigly when on previous steps it's mean buckling have occurred. So researching of the buckling area in this way could give the critical load with specified accuracy.

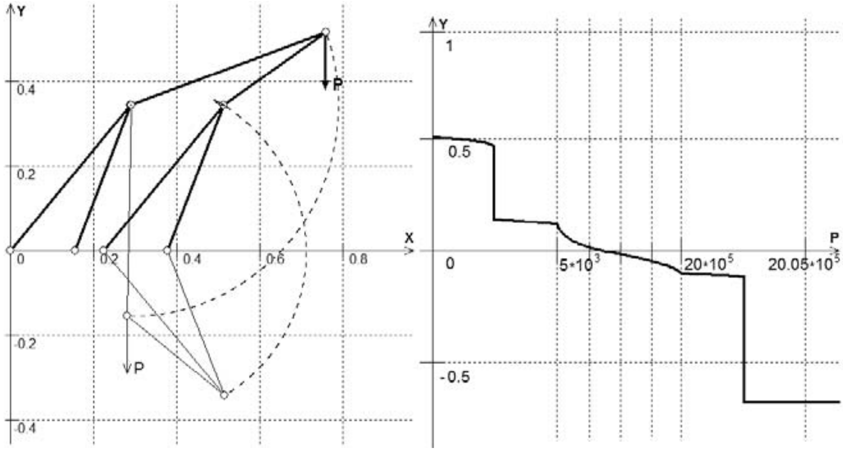

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Fig. 5. Complex rod construction after second buckling: a) general view; b) dependence of the displacement from the applied force what includes first and second buckling.

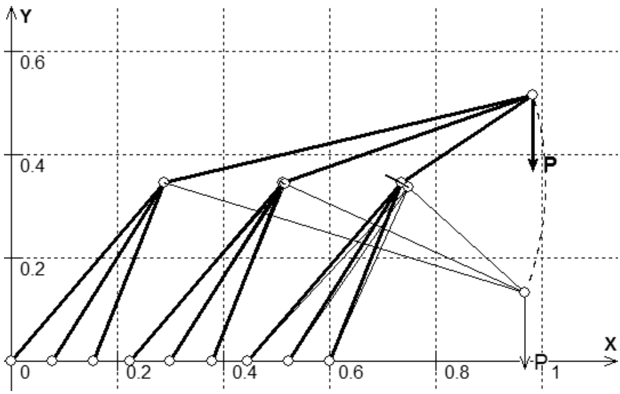

Fig. 6. First buckling in complex rod construction.

Conclusions. The conducted research shows ability to find critical loads for constrictions of any form what modeled by rods and in where buckling could take place.

During study of this phenomenon the graphics of applied force and displacement dependences have been plotted and stress–strain states of construction both before and after buckling have been found.

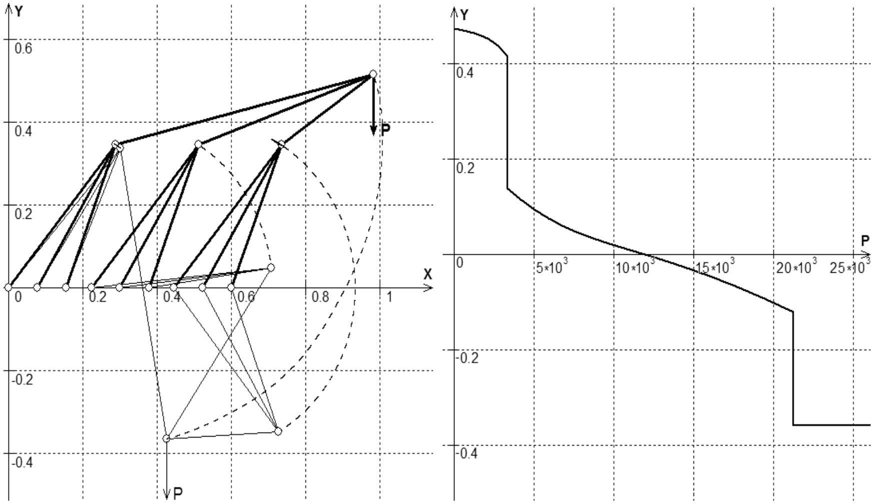

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Fig. 7. Complex rod construction after second buckling: a) general view; b) dependence of the displacement from the applied force what includes first and second buckling.

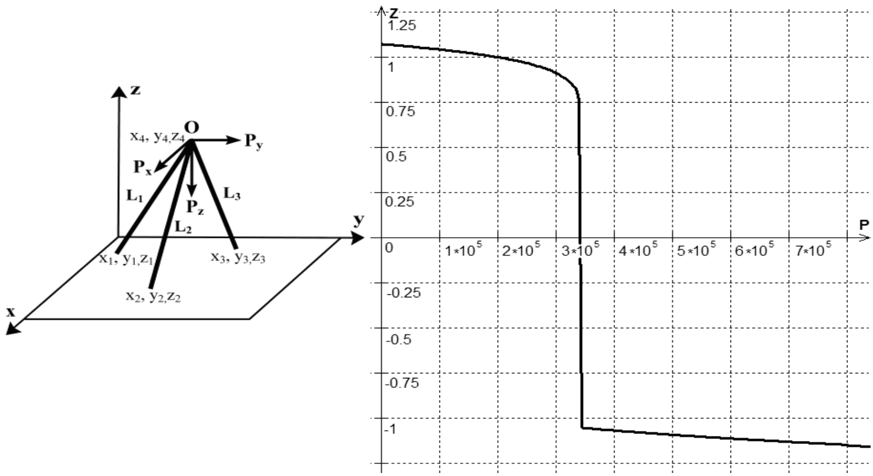

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Fig. 8. Simple three-dimensional rod construction after buckling: a) general view; b) dependence of the displacement from the applied force.

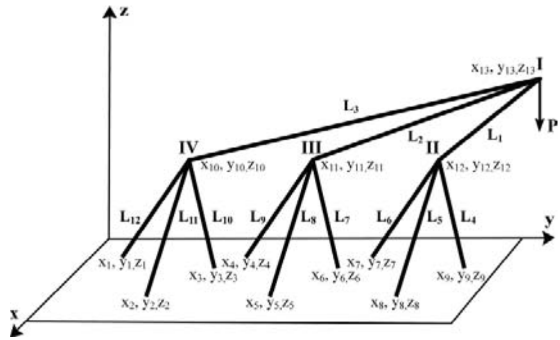

Fig. 9. Complex three-dimensional construction what have elements with different stiffness.

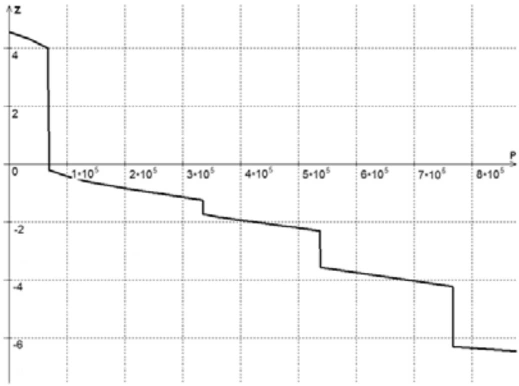

Fig. 10. Dependence of the displacement from the applied force showing what construction has four buckling appears.

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