3 D FOURIER COEFFICIENTS AND INTERFLATATION OF FUNCTIONS

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Modern problems of multivariate digital signal processing need solutions with new forms of data. It means that information about function is set of traces on the flats or set of traces on the lines or set of values on knots. The theory of interlineations and interflatation of functions is the most effective in this case [1], [2]. The first results of calculation of 2 D Fourier coefficients with using operators of spline–interlineation in case when information about a function is set of values on knots were presented in [4]. The main results of this investigation you can see in [4]. In [2], [5] the evaluating of two dimensions of Fourier coefficients with using operators of spline–interlineation on the lines of rectangulation was described.

The calculation of 3 D Fourier coefficients with using operators of spline–interflatation (case when information about function is set of values on knots) was studied in [6]. The evaluating of multivariate dimensions of Fourier coefficients also was considered in [7]–[9]. The main point of this research was rapid approximation, but not the new forms of data. The question of construction of the cubature formula of evaluating of 3 D Fourier coefficients (in case when information about nonoscillatory multiplier $f(x,y,z)$ is the set of traces on the lines) was not investigated.

The first idea of this work is representing and investigation of cubature formulas of approximate calculation of 3 D Fourier coefficients

$$I_1^3(m,n,p) = \int_0^1\int_0^1\int_0^1 f(x,y,z)\sin 2\pi mx \sin 2\pi ny \sin 2\pi pz dxdydz,$$

$$I_2^3(m,n,p) = \int_0^1\int_0^1\int_0^1 f(x,y,z)\cos 2\pi mx \cos 2\pi ny \cos 2\pi pz dxdydz,$$

$$I_3^3(m,n,p) = \int_0^1\int_0^1\int_0^1 f(x,y,z)e^{-i2\pi mx}e^{-i2\pi ny}e^{-i2\pi pz}dxdydz,$$

on the class of real functions of three variables, which are determinated on the domain $G=[0,1]^3$ with conditions $\left|f^{(r,r,0)}(x,y,z)\right| \le \bar{M}$, $\left|f^{(r,0,r)}(x,y,z)\right| \le \bar{M}$, $\left|f^{(0,r,r)}(x,y,z)\right| \le \bar{M}$, $\left|f^{(r,r,r)}(x,y,z)\right| \le \tilde{M}$, $r=1,2$.

The second idea of this work is to get the estimations of error of approaching of receiving cubature formulas.

1. Operators of Spline–interlineation and Spline–interflstation. Suppose that

$$h_{10}(x)=\begin{cases}\dfrac{x-x_1}{-\Delta}, & x_0\le x<x_1,\\ 0, & x\ge x_1,\end{cases}\qquad h_{1\ell}(x)=\begin{cases}0, & x\le x_{\ell-1},\\ \dfrac{x-x_\ell}{\Delta}, & x_{\ell-1}<x\le x_\ell,\end{cases}$$

$$h_{1k}(x)=\begin{cases}0, & x\le x_{k-1},\\ \dfrac{x-x_k}{\Delta}, & x_{k-1}<x<x_k,\\ \dfrac{x-x_{k+1}}{-\Delta}, & x_k\le x<x_{k+1},\\ 0, & x\ge x_{k+1},\end{cases}\qquad k=\overline{1,\ell-1},\quad x_k=k\Delta,\quad \Delta=\frac{1}{\ell},$$

$$\tilde h_{10}(x)=\begin{cases}\dfrac{x-\tilde x_1}{-\Delta_1}, & \tilde x_0\le x<\tilde x_1,\\ 0, & x\ge \tilde x_1,\end{cases}\qquad \tilde h_{1\ell^{3/2}}(x)=\begin{cases}0, & x\le \tilde x_{\ell^{3/2}-1},\\ \dfrac{x-x_\ell}{\Delta}, & \tilde x_{\ell^{3/2}-1}<x\le \tilde x_{\ell^{3/2}},\end{cases}$$

$$\tilde h_{1\tilde k}(x)=\begin{cases}0, & x\le \tilde x_{\tilde k-1},\\ \dfrac{x-\tilde x_{\tilde k}}{\Delta_1}, & \tilde x_{\tilde k-1}<x<\tilde x_{\tilde k},\\ \dfrac{x-\tilde x_{\tilde k+1}}{-\Delta_1}, & \tilde x_{\tilde k}\le x<\tilde x_{\tilde k+1},\\ 0, & x\ge \tilde x_{\tilde k+1},\end{cases}$$

$$\tilde k=\overline{1,\ell^{3/2}-1},\quad \tilde x_{\tilde k}=\tilde k\Delta_1,\qquad \Delta_1=\frac{1}{\ell^{3/2}}.$$

Next functions are determinated in the same way

1. $h_{2j}(y),\quad j=\overline{0,\ell},\quad h_{3s}(z),\quad s=\overline{0,\ell},\quad y_j=j\Delta,\quad z_s=s\Delta,\quad \Delta=\frac{1}{\ell};$
2. $\tilde h_{2\tilde j}(y)\quad \tilde j=\overline{0,\ell^{3/2}},\ \tilde h_{3\tilde s}(z)\quad \tilde s=\overline{0,\ell^{3/2}},\ \tilde y_{\tilde j}=\tilde j\Delta_1,\quad \tilde z_{\tilde s}=\tilde j\Delta_1,\quad \Delta_1=\frac{1}{\ell^{3/2}}.$

Let's see operators

$$O_1f(x,y,z)=\sum_{k=0}^{\ell}f(x_k,y,z)h_{1k}(x)\quad,\ O_2f(x,y,z)=\sum_{j=0}^{\ell}f(x,y_j,z)h_{2j}(y)\quad,$$

$$O_3f(x,y,z)=\sum_{s=0}^{\ell}f(x,y,z_s)h_{3s}(z),$$

$$\tilde O_1f(x,y,z)=\sum_{\tilde k=0}^{\ell^{3/2}}f(\tilde x_{\tilde k},y,z)h_{1\tilde k}(x)\quad,\ \widetilde{O_2}\,f(x,y,z)=\sum_{\tilde j=0}^{\ell^{3/2}}f(x,\tilde y_{\tilde j},z)\tilde h_{2\tilde j}(y)\quad,$$

$$\tilde O_3f(x,y,z)=\sum_{\tilde s=0}^{\ell^{3/2}}f(x,y,\tilde z_{\tilde s})\tilde h_{3\tilde s}(z).$$

Definition. By traces of function $f(x,y,z)$ on the lines we understand

$$f(x_k,y_j,z),\quad 0\le z\le 1,\ f(x_k,y,z_s),\ 0\le y\le 1,\ f(x,y_j,z_s),\ 0\le x\le 1.$$

Lemma 1. [1] *Operator of spline–inferflatation*

$$Of(x,y,z)=O_1f(x,y,z)+O_2f(x,y,z)+O_3f(x,y,z)-$$
$$-O_1O_2f(x,y,z)-O_2O_3f(x,y,z)-O_1O_3f(x,y,z)+O_1O_2O_3f(x,y,z)$$

has property $\left|f(x,y,z)-Of(x,y,z)\right|=O\left(\frac{1}{\ell^{3r}}\right)$.

Lemma 2. [1] *Operator of spline–interlineation basing on the inferflatation of functions*

$$\tilde{O}f(x,y,z)=O_1\tilde{O}_2f(x,y,z)+O_1\tilde{O}_3f(x,y,z)-$$
$$-O_1\tilde{O}_2\tilde{O}_3f(x,y,z)+O_2\tilde{O}_1f(x,y,z)+O_2\tilde{O}_3f(x,y,z)-$$
$$-O_2\tilde{O}_1\tilde{O}_3f(x,y,z)+O_3\tilde{O}_1f(x,y,z)+O_3\tilde{O}_2f(x,y,z)-O_3\tilde{O}_1\tilde{O}_2f(x,y,z)-$$
$$-O_1O_2f(x,y,z)-O_1O_3f(x,y,z)-O_2O_3f(x,y,z)+O_1O_2O_3f(x,y,z)$$

has property $\left|f(x,y,z)-\tilde{O}f(x,y,z)\right|=O\left(\frac{1}{\ell^{3r}}\right)$.

Let

$$G_{1k}(x,\xi,r)=\begin{cases}\dfrac{x_{k+1}-x}{x_{k+1}-x_k}\dfrac{(x_k-\xi)^{r-1}}{(r-1)!}, & x_k<\xi<x,\\[2ex] \dfrac{x_k-x}{x_{k+1}-x_k}\dfrac{(x_{k+1}-\xi)^{r-1}}{(r-1)!}, & x<\xi<x_{k+1},\end{cases}$$

$$\tilde{G}_{1\tilde{k}}(x,\xi,r)=\begin{cases}\dfrac{\tilde{x}_{\tilde{k}+1}-x}{\tilde{x}_{\tilde{k}+1}-\tilde{x}_{\tilde{k}}}\dfrac{(\tilde{x}_{\tilde{k}}-\xi)^{r-1}}{(r-1)!}, & \tilde{x}_{\tilde{k}}<\xi<x,\\[2ex] \dfrac{\tilde{x}_{\tilde{k}}-x}{\tilde{x}_{\tilde{k}+1}-\tilde{x}_{\tilde{k}}}\dfrac{(x_{k+1}-\xi)^{r-1}}{(r-1)!}, & x<\xi<\tilde{x}_{\tilde{k}+1},\end{cases}$$

and functions $G_{2j}(y,\eta,r)$, $\tilde{G}_{2\tilde{j}}(y,\eta,r)$, $G_{3s}(z,\varsigma,r)$, $\tilde{G}_{3\tilde{s}}(z,\varsigma,r)$, $r=1,2$ are determinated in the same way.

Lemma 3. [1] *The next equals are true*

$$f(x,y,z)-Of(x,y,z)=$$
$$\int_{x_k}^{x_{k+1}}\int_{y_j}^{y_{j+1}}\int_{z_s}^{z_{s+1}} f^{(r,r.r)}(\xi,\eta,\varsigma)G_{1k}(x,\xi,r)G_{2j}(y,\eta,r)G_{3s}(z,\varsigma,r)d\xi d\eta d\zeta\,;$$

$$\left(O_1 - O_1\tilde{O}_2 - O_1\tilde{O}_3 + O_1\tilde{O}_2\tilde{O}_3\right) f\left(x,y,z\right) =$$

$$\int_{\tilde{y}_{\tilde{j}}}^{\tilde{y}_{\tilde{j}+1}} \int_{\tilde{z}_{\tilde{s}}}^{\tilde{z}_{\tilde{s}+1}} f^{(0,r,r)}(x_k,\eta,\zeta)\tilde{G}_{2_{\tilde{j}}}(y,\eta,r)\tilde{G}_{3\tilde{s}}(z,\zeta,r)d\eta d\zeta \ ;$$

Lemma 4. [10] *The next estimations are true*

$$\int_{x_k}^{x_{k+1}} \int_{x_k}^{x_{k+1}} \left|G_1\left(x,\xi,r\right)\right| d\xi dx \le \frac{2\Delta^{r+1}}{(r+2)!}, \int_{y_j}^{y_{j+1}} \int_{y_j}^{y_{j+1}} \left|G_{2j}\left(y,\eta,r\right)\right| d\eta dy \le \frac{2\Delta^{r+1}}{(r+2)!},$$

$$\int_{z_s}^{z_{s+1}} \int_{z_s}^{z_{s+1}} \left|G_{3s}\left(z,\varsigma,r\right)\right| d\varsigma dz \le \frac{2\Delta^{r+1}}{(r+2)!}.$$

2. The Cubature Formulas of the Evaluating of 3 D Fourier coefficients with Using the Interflatation of Functions. For evaluating of integrals $I_\mu^3(m,n,p), \quad \mu = 1,2,3$ the following formulas are suggested:

$$\tilde{\Phi}_1^3\left(m,n,p\right) = \int_0^1 \int_0^1 \int_0^1 \tilde{O}f(x,y,z)\sin 2\pi mx \sin 2\pi ny \sin 2\pi pz dxdydz \ ,$$

$$\tilde{\Phi}_2^3\left(m,n,p\right) = \int_0^1 \int_0^1 \int_0^1 \tilde{O}f(x,y,z)\cos 2\pi mx \ \cos 2\pi ny \ \cos 2\pi pz dxdydz \ ,$$

$$\tilde{\Phi}_3^3\left(m,n,p\right) = \int_0^1 \int_0^1 \int_0^1 \tilde{O}f(x,y,z)e^{-i2\pi mx}e^{-i2\pi ny}e^{-i2\pi pz}dxdydz \, .$$

Theorem. *For cubature formula* $\tilde{\Phi}_1^3\left(m,n,p\right)$ *of evaluating* $I_1^3\left(m,n,p\right)$ *the next estimation is true:*

$$\left|I_1^3(m,n,p) - \tilde{\Phi}_1^3\left(m,n,p\right)\right| \le \left(\frac{8\tilde{M}}{\left[(r+2)!\right]^3} + \frac{12\bar{M}}{\left[(r+2)!\right]^2}\right)\frac{1}{\ell^{3r}} \, .$$

Proof. Let's get the estimation of calculation

$$\left|I_1^3(m,n,p) - \tilde{\Phi}_1^3\left(m,n,p\right)\right| \le$$

$$\le \left|I_1^3(m,n,p) - \Phi_1^3\left(m,n,p\right)\right| + \left|\Phi_1^3(m,n,p) - \tilde{\Phi}_1^3\left(m,n,p\right)\right| \le$$

$$\le \int_0^1 \int_0^1 \int_0^1 \left|f(x,y,z) - Of(x,y,z)\right| dxdydz +$$

$$+ \int_0^1 \int_0^1 \int_0^1 \left|Of(x,y,z) - \tilde{O}f(x,y,z)\right| dxdydz =$$

$$= I_1 + I_2 \, .$$

Consider the estimation of the first component

$$I_1 \le \sum_{k=0}^{\ell-1}\sum_{j=0}^{\ell-1}\sum_{s=0}^{\ell-1} \int_{x_k}^{x_{k+1}} \int_{y_j}^{y_{j+1}} \int_{z_s}^{z_{s+1}} \int_{x_k}^{x_{k+1}} \int_{y_j}^{y_{j+1}} \int_{z_s}^{z_{s+1}} \left|f^{(r,r.r)}(\xi,\eta,\varsigma)\right| \times$$

$$\times\left|G_{1k}\left(x,\xi,r\right)\right|\left|G_{2j}\left(y,\eta,r\right)\right|\left|G_{3s}\left(z,\varsigma,r\right)\right|d\xi d\eta d\zeta dxdydz \le$$

$$\le \tilde{M}\sum_{k=0}^{\ell-1}\sum_{j=0}^{\ell-1}\sum_{s=0}^{\ell-1}\frac{2\Delta^{r+1}}{(r+2)!}\frac{2\Delta^{r+1}}{(r+2)!}\frac{2\Delta^{r+1}}{(r+2)!}=\tilde{M}\frac{8\Delta^{3r}}{\left[(r+2)!\right]^3}=\frac{8\tilde{M}}{\left[(r+2)!\right]^3\ell^{3r}}.$$

Consider the estimation of the second component

$$I_2=\int_0^1\int_0^1\int_0^1\left|O_1f(x,y,z)+O_2f(x,y,z)+O_3f(x,y,z)-\right.$$

$$-O_1O_2f(x,y,z)-O_2O_3f(x,y,z)-O_1O_3f(x,y,z)+O_1O_2O_3f(x,y,z)-$$

$$-O_1\tilde{O}_2f(x,y,z)-O_1\tilde{O}_3f(x,y,z)+O_1\tilde{O}_2\tilde{O}_3f(x,y,z)-O_2\tilde{O}_1f(x,y,z)-$$

$$-O_2\tilde{O}_3f(x,y,z)+O_2\tilde{O}_1\tilde{O}_3f(x,y,z)-O_3\tilde{O}_1f(x,y,z)-O_3\tilde{O}_2f(x,y,z)+$$

$$+O_3\tilde{O}_1\tilde{O}_2f(x,y,z)+O_1O_2f(x,y,z)+O_1O_3f(x,y,z)+$$

$$\left.+O_2O_3f(x,y,z)-O_1O_2O_3f(x,y,z)\right|dxdydz=$$

$$=\int_0^1\int_0^1\int_0^1\left|O_1f(x,y,z)+O_2f(x,y,z)+O_3f(x,y,z)-\right.$$

$$-O_1\tilde{O}_2f(x,y,z)-O_1\tilde{O}_3f(x,y,z)+O_1\tilde{O}_2\tilde{O}_3f(x,y,z)-O_2\tilde{O}_1f(x,y,z)-$$

$$-O_2\tilde{O}_3f(x,y,z)+O_2\tilde{O}_1\tilde{O}_3f(x,y,z)-O_3\tilde{O}_1f(x,y,z)-$$

$$\left.-O_3\tilde{O}_2f(x,y,z)+O_3\tilde{O}_1\tilde{O}_2f(x,y,z)\right|dxdydz\le$$

$$=\int_0^1\int_0^1\int_0^1\left|\left(O_1-O_1\tilde{O}_2-O_1\tilde{O}_3+O_1\tilde{O}_2\tilde{O}_3\right)f(x,y,z)\right|dxdydz+$$

$$+\int_0^1\int_0^1\int_0^1\left|\left(O_2-O_2\tilde{O}_1-O_2\tilde{O}_3+O_2\tilde{O}_1\tilde{O}_3\right)f(x,y,z)\right|dxdydz+$$

$$+\int_0^1\int_0^1\int_0^1\left|\left(O_3-O_3\tilde{O}_1-O_3\tilde{O}_2+O_3\tilde{O}_1\tilde{O}_2\right)f(x,y,z)\right|dxdydz\le$$

$$\le\sum_{k=0}^{\ell-1}\sum_{\tilde{j}=0}^{\ell^{3/2}-1}\sum_{\tilde{s}=0}^{\ell^{3/2}-1}\int_{x_k}^{x_{k+1}}\int_{\tilde{y}_{\tilde{j}}}^{\tilde{y}_{\tilde{j}+1}}\int_{\tilde{z}_{\tilde{s}}}^{\tilde{z}_{\tilde{s}+1}}\int_{\tilde{y}_{\tilde{j}}}^{\tilde{y}_{\tilde{j}+1}}\int_{\tilde{z}_{\tilde{s}}}^{\tilde{z}_{\tilde{s}+1}}\left|f^{(0,r,r)}(x_k,\eta,\zeta)\right|\times$$

$$\times\left|\tilde{G}_{2_{\tilde{j}}}(y,\eta,r)\right|\left|\tilde{G}_{3\tilde{s}}(z,\zeta,r)\right|d\eta d\zeta dxdydz+$$

$$+\sum_{j=0}^{\ell-1}\sum_{\tilde{k}=0}^{\ell^{3/2}-1}\sum_{\tilde{s}=0}^{\ell^{3/2}-1}\int_{\tilde{x}_{\tilde{k}}}^{\tilde{x}_{\tilde{k}+1}}\int_{y_j}^{y_{j+1}}\int_{\tilde{z}_{\tilde{s}}}^{\tilde{z}_{\tilde{s}+1}}\int_{\tilde{x}_{\tilde{k}}}^{\tilde{x}_{\tilde{k}+1}}\int_{\tilde{z}_{\tilde{s}}}^{\tilde{z}_{\tilde{s}+1}}\left|f^{(r,0,r)}(\xi,y_j,\zeta)\right|\times$$

$$\times\left|\tilde{G}_{1\tilde{k}}(x,\xi,r)\right|\left|\tilde{G}_{3\tilde{s}}(z,\zeta,r)\right|d\xi d\zeta dxdydz+$$

$$+\sum_{s=0}^{\ell-1}\sum_{\tilde{k}=0}^{\ell^{3/2}-1}\sum_{\tilde{j}=0}^{\ell^{3/2}-1}\int_{\tilde{x}_{\tilde{k}}}^{\tilde{x}_{\tilde{k}+1}}\int_{\tilde{y}_{\tilde{j}}}^{\tilde{y}_{\tilde{j}+1}}\int_{z_s}^{z_{s+1}}\int_{\tilde{x}_{\tilde{k}}}^{\tilde{x}_{\tilde{k}+1}}\int_{\tilde{y}_{\tilde{j}}}^{\tilde{y}_{\tilde{j}+1}}\left|f^{(r,r,0)}(\xi,\eta,z_s)\right|\times$$

$$\times\left|\tilde{G}_{1\tilde{k}}(x,\xi,r)\right|\left|\tilde{G}_{2\tilde{j}}(y,\eta,r)\right|d\xi d\eta dxdydz\le$$

$$=\bar{M}\ell\Delta\ell^{3/2}\ell^{3/2}\frac{2\Delta_1^{r+1}}{(r+2)!}\frac{2\Delta_1^{r+1}}{(r+2)!}+\bar{M}\ell\Delta\ell^{3/2}\ell^{3/2}\frac{2\Delta_1^{r+1}}{(r+2)!}\frac{2\Delta_1^{r+1}}{(r+2)!}+$$

$$+\bar{M}\ell\Delta\ell^{3/2}\ell^{3/2}\frac{2\Delta_1^{r+1}}{(r+2)!}\frac{2\Delta_1^{r+1}}{(r+2)!}=$$

$$=12\bar{M}\frac{\Delta_1^{r}}{(r+2)!}\frac{\Delta_1^{r}}{(r+2)!}=\frac{12\bar{M}}{\left[(r+2)!\right]^2}\Delta_1^{2r}=$$

$$=\frac{12\bar{M}}{\left[(r+2)!\right]^2}\left(\frac{1}{\ell^{3/2}}\right)^{2r}=\frac{12\bar{M}}{\left[(r+2)!\right]^2}\frac{1}{\ell^{3r}}.$$

At last, $\left|I_1^3(m,n,p)-\tilde{\Phi}_1^3\left(m,n,p\right)\right|\le\left(\frac{8\tilde{M}}{\left[(r+2)!\right]^3}+\frac{12\bar{M}}{\left[(r+2)!\right]^2}\right)\frac{1}{\ell^{3r}}$. Theorem is proved.

3. Numerical experiment. In the table 1 for the function $f\left(x,y,z\right)=\sin(x+y+z)$ the results of approximate calculation of integral $I_1^3(m,n,p)$ are presented for different m,n,p by cubature formula $\tilde{\Phi}_1^3(m,n,p)$. Let $\varepsilon_1=\left|I_1^3(m,n,p)-\tilde{\Phi}_1^3(m,n,p)\right|$, $\varepsilon_2=\left(\frac{8\tilde{M}}{\left[(r+2)!\right]^3}+\frac{12\bar{M}}{\left[(r+2)!\right]^2}\right)\frac{1}{\ell^{3r}}$. In this case $\tilde{M}=\bar{M}=1,\quad r=2$ and $\varepsilon_2=0.021\cdot\frac{1}{\ell^6}$. Exact values of integrals are:

$$I_1^3(1,2,3)=0.000043384641833,\quad I_1^3(4,4,4)=0.000003946808581.$$

Table 1. Results of the calculation of integral $I_1^3(m,n,p)$

m	n	p	ℓ	$\tilde{\Phi}_1^3(m,n,p)$	ε_1	ε_2
1	2	3	4	0.000043384384762	$2.5\cdot10^{-10}$	$5.1\cdot10^{-6}$
			9	0.000043384640066	$1.7\cdot10^{-12}$	$3.9\cdot10^{-8}$
			16	0.000043384641777	$5.5\cdot10^{-14}$	$1.2\cdot10^{-9}$
			25	0.000043384641829	$3.6\cdot10^{-15}$	$8.6\cdot10^{-11}$
4	4	4	9	0.000003946808415	$1.6\cdot10^{-13}$	$1.2\cdot10^{-9}$
			16	0.000003946808577	$4.9\cdot10^{-15}$	$1.2\cdot10^{-9}$

Conclusions. In this work cubature formulas of the evaluating of 3 D Fourier coefficients with using operators of spline–interflatation are built and investigated on the class of real functions on the domain $G=[0,1]^3$ and $\left|f^{(r,r,0)}(x,y,z)\right|\le\bar{M}$, $\left|f^{(r,0,r)}(x,y,z)\right|\le\bar{M}$, $\left|f^{(0,r,r)}(x,y,z)\right|\le\bar{M}$, $\left|f^{(r,r,r)}(x,y,z)\right|\le\tilde{M}$, $r=1,2$. Information about function is set of it's traces on the perpendicular lines. The estimations of error of approaching of the cubature formulas are presented. Numerical example proved theoretical results.

Next step of our research is proving of the optimal by the order of exactness of this cubature formula.

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