SELF-ORGANIZATION IN MAGNETC FLUIDS

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Interaction between the magnetic dipoles of ferroparticles in the ferrofluids may cause the reversible formation and destruction of aggregates (clusters) of magnetic particles with a change of the magnetic field and temperature. Each aggregate is characterized by a quantity of ferroparticles in it and intrinsic magnetic moment. In essence, the ferrofluid transforms into a state with a new microstructure, which differs from the state of the interacting single ferro-particles presence of clusters containing a finite number of ferroparticles. The fact that by means the magnetic field it is possible to manage the internal structure of a magnetizable medium, provides the ability to create new tech-nological nanomaterials. Therefore, the magnetic fluids with varying microstructure appear to be the object of intensive study. In recent years a some number of papers which describe experiments with films of magnetizable fluids (ferrofluids) have appeared [1–5].

Under the influence of an external magnetic field ferroparticles included in the composition of the liquid to form regular ordered configuration, the structure is determined by the direction and magnitude of the field, the liquid becomes anisotropic and, therefore, changed the terms of its interaction with optical radiation. The dependence of the optical properties of magnetizable fluids from the magnetic field opens up a new area of their practical use, namely, magneto [2]. In connection with possible applications the magneto seem very relevant experimental and theoretical studies of processes occurring with ferroparticles in fluid under the action of an external field, and mathematical models describing these processes. A detailed list of published works and their characteristics are available in [6], devoted to the study of patterns of condensation ferroparticles in the absence of the field. Structural transformations in ferrofluids in the fields of varying intensity are discussed in a recent paper [7–10]. In [7] columnar configuration of ferroparticles in fluid films theoretically investigated, estimates for the geometric parameters of hexagonal structures in the field orthogonal to the film derived. The authors of [8] studied the dependence of the static magnetic properties of ferrofluids on the size and concentration of particles, the dependence of the magnetization curve of the form on the presence of fluid in the chain aggregates. In [9] the results of studying the behavior of the ferrofluid in a strong magnetic field, within the mean–field theory shows that at temperatures below some critical value, the second order phase transition in liquid form layered structures. In [10] analyzed the influence of the thickness of the liquid in the form appearing in its spatial structure.

A feature of these papers is introduced by the authors suggested the presence of a ferrofluid formed aggregates of magnetic particles. Directly processes the origin and the dynamics of aggregates in them are not investigated. The simplest model that takes into account the dipole–dipole interaction condition ferroparticles in self-consistent field and the possibility of formation of these chain aggregates was proposed in [11]. The equilibrium magnetization of the medium with varying microstructure is determined by two additional parameters – the average number of particles in aggregates $\gamma = \nu^{-1}$ and the effective magnetic field parameter λ, which takes into account the field produced by the particles. The origin and destruction of aggregates, i.e. changes in the microstructure of the fluid is regarded as second order phase transition and describe the system of two quasilinear equations of parabolic type with respect to λ and ν. The equilibrium magnetization curve of the medium studied in [13, 14], where conditions are obtained for the existence of wave modes such as switching waves. In [15] presented a qualitative analysis of the system in the limiting case where the diffusion phenomena in the liquid can be neglected. In [16–20] examined the spatial and spatio–temporal structures such as static and moving solitons and periodic striations, which are formed in a liquid in a constant homogeneous magnetic field. This work contains the results of the study of spatial structures that describe the redistribution ferroparticles in a magnetizable fluid at rest, observed in experiments.

1. Changes in the microstructure of magnetic fluids.

1.1. Mean-field approximation in magnetizable media [28, 29]. To describe the processes of structuring let us analyze the mean-field approximation in polydispersive liquid magnetics.

The magnetization of the non-ideal media is determined by the equation

$$M = M^{(0)}\left(\rho, T, H_e, c_1, ..., c_n\right), \; H_e = H + \lambda M, \; M_{H=0} = 0, \qquad (1.1)$$

λ, c_i – stand for the effective (mean) field parameter and component concentrations respectively. For λ =0, $M^{(0)}$ determines the magnetization of an ideal medium. In the case where the magnetic properties has only one component, the function $M^{(0)}$ is represented by Langevin relation

$$M^{(0)} = m_1 n_1 L\left(\xi_e\right), \quad \xi_e = \frac{m_1\left(H + \lambda M\right)}{kT}, \qquad L = \mathrm{cth}\xi_e - \xi_e^{-1}, \qquad (1.2)$$

m_1 and n_1 are magnetic moment and the number of separate ferroparticles per unit volume respectively.

Hence, depending on the values of the effective field there follow the classical theories of magnetization from (1.2):

when $\lambda = \frac{4\pi}{3}$ and $\xi_e << 1$ (in the case of weak fields) – the law of magnetization of the Clausius–Mosotti

$$\frac{\mu - 1}{\mu + 2} = \frac{\sigma_1}{3}; \quad \sigma_1 = \frac{4\pi}{3}\frac{m_1^2 \rho c_1}{MkT},$$

M is molecular mass; when

$$\lambda = \frac{8\pi}{2+3\sigma_1+(9+6\sigma_1+9\sigma_1^2)^{1/2}}, \quad \xi_e << 1$$

– the equation of the magnetic state of the Debye–Onsager in the form of

$$(\mu-1)(2\mu+1)=3\sigma_1\mu.$$

Using the theory of Leontovich–Mandelstam nonequilibrium thermodynamic processes [30] it can be shown that in the equilibrium thermodynamic states for the parameter of the effective field the most general dependence on the governing parameters is of the form

$$\lambda = \lambda^{(0)}\left(\rho, T, M^2, c_1, ..., c_n\right). \tag{1.3}$$

Thus, the parameter of the effective field depends on the magnetic field through the square of the magnetization.

It is shown that magnetic–dipole interaction between ferroparticle make contribution into the internal energy of the magnetic fluid and the magnetostrictive stresses in magnetizable media. It should be noted [25] that taking into account only the polydispersity of magnetic fluids cannot be explained the separation of the liquid on the phases in a magnetic field. Magnetic fluids should be considered as real microheterogeneous colloids, because of stratification of the solutions with a magnetic particles in a weak magnetic field in the presence of the dipole interaction.

1.2. The equilibrium thermodynamic model of structuring of magnetic fluids. In the presence of aggregates the ferrofluid is proposed to describe in the "mean representation" considering the magnetic fraction in the form of aggregates (clusters), each of which is combined by $\gamma = \nu^{-1}$ ferroparticles. The number n_a of clusters per volume unit and magnetic moment m_a are determined by the equations: $n_a = \nu\, n_1, m_a = \nu^{-1} m_1$. Therefore, the magnetization of a structured medium is determined by the equation

$$M = m_1 n_1 L(\xi), \xi = \frac{m_1(H+\lambda M)}{kT\nu}, \qquad L = \mathrm{cth}\,\xi - \xi^{-1} \tag{1.4}$$

The general dependence (1.3) is chosen in the form of:

$$M^2 = a_0 + a_1\lambda + a_2\lambda^2 + a_3\lambda^3, a_i = a_i(n_1, T)$$

Under certain restrictions on the coefficients a_i of the graph $M^2 = M^2(\lambda)$ is as shown in Fig. 1, and $0 < \tilde{M}_2 < \tilde{M}_1 < m_1 n_1$.

When changing the magnetic field the mean-field parameter λ abruptly changes from values $\lambda^{(1)}$ on the upper branch OA (unstructured phase) to the values $\lambda^{(2)}$ on the branch BC (structured phase) at the value $M = M_*$ of the magnetization structuring. It should be noted [28] that the first derivatives of the thermodynamic potential does not change as the same are the magnetization and the magnetic phase concentration at value $M = M_*$. Thus, the structuring of the magnetic fluid is implemented as a result of phase transition of the second kind. The average number of ferromagnetic particles in the resulting aggregates equals

$$\gamma = \nu^{-1}(n_1, T) = \left(1 + \frac{m_1 M_*}{kT\xi_e} < \lambda >\right)^{-1}; < \lambda >= \lambda_*^{(2)} - \lambda_*^{(1)} < 0; \qquad (1.5)$$

Asterisk indicates values at the point of structuring. The critical field H_* of the structuring is determined by the relationship (1.4) as

$$H_* = H_*(n_1, T) = -\lambda_*^{(1)} M_* + \frac{kT\xi_{e*}}{m_1}; \quad \xi_{e*} = \xi_{e*}(n_1, T) = L^{-}\left(\frac{M_*}{m_1 n_1}\right).$$

where L^{-} is the inverse Langevin function.

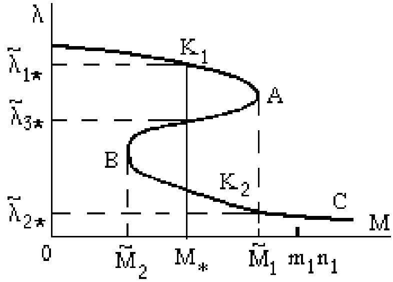


Fig. 1. Dependence of the effective field parameter on the magnetization in the magnetic fluid with transformed microstructure

The value H_* increases with temperature and decreases and decreases with increasing of volume concentration ferroparticles. H_* depends linearly on the temperature, and T_* is a linear function of magnetic field strength. The point of transition of the ferrofluid in a structured state the corner points in the equilibrial dependences of M = M (H), M = M (T) correspond. At the same time $< M_T > / < M_H >= -k\xi_{e*} / m_1$. These features of the phase transition correspond to the experimental results ([31, 32, 33, 34]. The average number of associated ferroparticles in aggregates, as can be estimated by the formula (2.6), varies from two to twenty in order. It was shown [29] that structured phase is diffusively stable, but with increasing magnetic field strength in the range H> H* ferrofluid stratifies into the phases with large magnetic phase concentration and a non-magnetic phase in the liquid carrier in connection with violation of the conditions of the diffusion stability. Settings λ and ν play the role of the parameters of the order of the system. The constancy of the parameters λ and ν corresponds to a system with a constant microstructure.

1.3. Dynamic phase transitions. The complete system of equations for the dynamics of a magnetizable medium with transformed microstructure is obtained in [11]. In describing the process of structuring one can neglect the difference in the masses of ferroparticles and molecules of the liquid carrier. In addition, since the processes of structuring implements by microdiffusion of ferroparticles, let us consider in the follows the fluid velocity is zero. Then from among the governing parameters can be excluded ferrofluid density and velocity. The general system for the structuring processes includes balance equations for the order parameters λ and ν, the equations of electrodynamics and the entropy balance equation in the form:

$$\frac{d\lambda}{dt} = L_{66}\left[\rho Q^{(\lambda)} + \alpha_1 \nabla\left(\rho \nabla \lambda\right)\right]; \ \frac{d\nu}{dt} = L_{88}\left[\rho Q^{(\nu)} + \alpha_2 \nabla\left(\rho \nabla \nu\right)\right] \qquad (1.6)$$

$$\mathrm{div}\vec{B} = 0 \ ; \ \mathrm{rot}\vec{H} = 0 \ ; \ \vec{B} = \vec{H} + 4\pi\vec{M} \ ;$$

$$M = m_1 n_1 L(\xi) \ ; \ \xi = \frac{m_1\left(H + \lambda M\right)}{kT\nu} \ ;$$

$$\rho T \frac{ds}{dt} = \mathrm{div}\left(\kappa \nabla T\right) + L_{66}\left[\rho Q^{(\lambda)} + \alpha_1 \nabla\left(\rho \nabla \lambda\right)\right]^2 + L_{88}\left[\rho Q^{(\nu)} + \alpha_2 \nabla\left(\rho \nabla \nu\right)\right]^2 ;$$

Here it is written by neglecting of cross -effects, mutual diffusion and vector fluxes of quantities λ and ν. In accordance with the second law of thermodynamics the phenomenological coefficients κ, L_{66}, L_{88} are nonnegative. Volume density of the internal energy is given by

$$u = \frac{1}{2}\left[\alpha_1\left(\nabla\lambda\right)^2 + \alpha_2\left(\nabla\nu\right)^2\right] + u^{(0)}\left(\rho, T, \vec{B}\vartheta, \lambda, \nu\right),$$

The first term describes the energy of microinhomogeneities in accordance with the Ornstein–Zernike theory [35], due to the inhomogeneity of order parameters: $\alpha_1 \ge 0$, $\alpha_2 \ge 0$. The thermodynamic potential $f^{(0)} = u^{(0)} - Ts - \frac{BH\vartheta}{4\pi}$ satisfies the Gibbs equation

$$df^{(0)} = -sdT - \frac{BH\vartheta}{4\pi}dH - Q^{(\lambda)}d\lambda - Q^{(\nu)}d\nu$$

and defined as follows:

$$f^{(0)} = \tilde{f}^{(0)}\left(\rho, T, \lambda, \nu\right) - \frac{H^2}{8\pi\rho} + \frac{\lambda M^2}{2\rho} - \frac{n_1 kT\nu}{\rho}\ln\frac{\mathrm{sh}\xi_e}{\xi_e} \qquad (1.7)$$

Upon receipt of this formula for all values λ and ν of the field in the regions of their changes the condition of paramagnetic states is assumed, which has the form: $T > T_k = \lambda m_1^2 n_1 / (3k\nu)$. The system of equations (1.6) is closed by the following equations of state of the environment with transformed microstructure:

$$s=-\frac{\partial\tilde{f}^{(0)}}{\partial T}+\frac{n_1 k\nu}{\rho}\left[\ln\frac{sh\xi_e}{\xi_e}-\xi_e L(\xi_e)\right], \qquad (1.8)$$

$$Q^{(\lambda)}=-\frac{\partial\tilde{f}^{(0)}}{\partial\lambda}+\frac{M^2}{2\rho};\; Q^{(\nu)}=-\frac{\partial\tilde{f}^{(0)}}{\partial\nu}+\frac{n_1 kT}{\rho}\left[\ln\frac{sh\xi_e}{\xi_e}-\xi_e L(\xi_e)\right].$$

In the case of a magnetizable medium, the microstructure of which changes as a result of the phase transition of type II, basing on the basic conceptions of the theory of phase transitions by Landau [36], we obtain for $\tilde{f}^{(0)}$

$$\tilde{f}^{(0)}=f^0(\rho,T)+\frac{\lambda M_*^2}{2\rho}+\frac{n_1 kT\nu}{\rho}\left[\ln\frac{sh\xi_{e*}}{\xi_{e*}}-\xi_{e*}L_*\right]+\Phi\left[\left(m_1^2 n_1^2/\rho\right)L_*\lambda\right]-$$
$$-\Phi\left[\frac{n_1 kT}{\rho}\xi_{e*}(\nu-\nu_{1*})+\frac{m_1^2 n_1^2}{\rho}L_*\lambda_{1*}\right], \qquad (1.9)$$

where the polynomial $\Phi(\omega)$ is defined as

$$\Phi(\omega)=A(\rho,T)\int(\omega-\omega_1)(\omega-\omega_2)(\omega-\omega_3)\,d\omega,$$

$$\omega_i=\left(m_1^2 n_1^2/\rho\right)L_*\lambda_{i*};\quad \nu_{i*}-\nu_{j*}=\frac{m_1^2 n_1}{kT}\frac{L_*}{\xi_{e*}}\left(\lambda_{i*}-\lambda_{j*}\right);\quad A<0$$

$$\lambda_{1*}\ge\lambda_{3*}\ge\lambda_{2*};\nu_{1*}\ge\nu_{3*}\ge\nu_{2*}.$$

The constructed function $f^{(0)}$ satisfies the necessary conditions at the phase transition II-nd kind $\xi_e=\xi_{e*}$

$$<f^{(0)}>=0;<p>=0;<s>=0;<M>=0.$$

Generalized thermodynamic forces $Q^{(\lambda)}$ and $Q^{(\nu)}$ have the form

$$Q^{(\lambda)}=-A\left(\frac{m_1^2 n_1^2}{\rho}L_*\right)^4(\lambda-\lambda_{1*})(\lambda-\lambda_{2*})(\lambda-\lambda_3)+\frac{M^2}{2\rho}-\frac{M_*^2}{2\rho},$$

$$Q^{(\nu)}=A\left(\frac{n_1 kT}{\rho}\xi_{e*}\right)^4(\nu-\nu_{1*})(\nu-\nu_{2*})(\nu-\nu_{3*})+\frac{n_1 kT}{\rho}\left[\ln\frac{\xi_{e*}sh\xi_e}{\xi_e sh\xi_{e*}}-\xi_e L+\xi_{e*}L_*\right].$$

The equilibrium relation $\lambda=\lambda^{(0)}\left(M^2\right)$ coincides with (1.3) in the case

$$|A|<\frac{64}{3}\left(\frac{\rho}{n_1 kT}\right)^3\left[\xi_{e*}L_*-\ln\frac{sh\xi_{e*}}{\xi_{e*}}\right]\Big/\left(\xi_{e*}^4|<\nu>|^3\right).$$

At equilibrium the average number of particles in the clusters, as well as the mean-field parameter depends on the magnetic field strength through the magnetization of the medium. If microstructure of magnetic fluid changes with the magnetic field, then in the absence of the field some number of ferroparticles are united in clusters.

Natural boundary conditions for the parameters λ and ν follow from the continuity of a normal component of the vector energy flux density on a solid surface Σ. Given the continuity of components of the stress vector $(< p_{ik} n_k >= 0)$, the normal component of the vector heat flux density ($< q_k n_k >= 0$), the tangential components of the electric and magnetic fields $\left(< \vec{H}_\tau >= 0, < \vec{E}_\tau >= 0\right)$ and the normal component of magnetic induction ($\left\langle \vec{B}_n \right\rangle = 0$), we obtain:

$$\left.\frac{\partial \lambda}{\partial n}\right|_\Sigma = 0; \left.\frac{\partial \nu}{\partial n}\right|_\Sigma = 0, \tag{1.10}$$

so that the diffusion parameters λ and ν across the border is missing.

It should be noted that the establishment of equilibrium distributions λ and ν parameters, as follows from (1.6), is accompanied by energy dissipation and redistribution of temperature in the bulk liquid. Assuming the heat capacity of the liquid carrier is large enough, we can neglect this heterogeneity, assuming a constant temperature. In this case, the magnetic field and temperature can be regarded as external parameters controlling the processes of change in the microstructure of the subsystem of magnetic particles in ferrofluid composition.

2. Spatial structure of the magnetizable fluid.

2.1. Statement of the problem. Consider a fixed amount of magnetizable liquid in a constant homogeneous magnetic field intensity $\vec{H}$. The equilibrium state of a liquid is determined by its density ρ, temperature T, magnetic field strength H, the average number of particles in aggregates $\gamma = \nu^{-1}$ and the parameter of the effective magnetic field λ. Change in the magnetic state of the medium is described by a system of equations (1.6) which reduces to:

$$\tau_\lambda \frac{\partial \lambda}{\partial t} = D_\lambda \Delta\lambda + Q^{(\lambda)}(\lambda, \nu; H), \quad \tau_\nu \frac{\partial \nu}{\partial t} = D_\nu \Delta\nu + Q^{(\nu)}(\lambda, \nu; H). \tag{2.1}$$

The functions $Q^{(\lambda)}(\lambda, \nu; H)$ and $Q^{(\nu)}(\lambda, \nu; H)$ are given by:

$$Q^{(\lambda)}(\lambda, \nu; H) = \frac{1}{2}\left[L^2 - L_*^2 - 2\beta(\lambda - \lambda_{1*})(\lambda - \lambda_{2*})(\lambda - \lambda_{3*})\right],$$

$$Q^{(\nu)}(\lambda, \nu; H) = f(\xi) - f(\xi_*) + \alpha(\nu - \nu_{1*})(\nu - \nu_{2*})(\nu - \nu_{3*}).$$

Here $\tau_\lambda = (L_{66} m_1^2 n_1^2)^{-1}$, $\tau_\nu = (L_{88} n_1 kT)^{-1}$, $D_\lambda = \alpha_1 \rho / (m_1^2 n_1^2)$, $D_\nu = \alpha_2 \rho / (n_1 kT)$, $\alpha = A\left(n_1 kT / \rho\right)^3 \xi_{e*}^4$, $\beta = A\left(m_1^2 n_1^2 / \rho\right)^3 L_*^4$, $M_s = m_1 n_1 -$ the saturation magnetization of the fluid, $n_1 = c_1 \rho / \mathcal{M} -$ the volume density ferroparticles, $c_1 -$ the mass concentration of particles, which is assumed to be constant, $\mathcal{M} -$ the mass of a single ferroparticles, $L_* := L(\xi_*) = M_* / M_s$, where

M_* – the magnetization structure of the liquid; λ_{i*}, ν_{i*} $(i=\overline{1,3})$ – equilibrium values λ and ν at $M=M_*$, $\lambda_{1*}>\lambda_{3*}>\lambda_{2*}$; $\lambda_{1*}+\lambda_{2*}=2\lambda_{3*}$; $\nu_{1*}>\nu_{3*}>\nu_{2*}$; $\nu_{i*}-\nu_{j*}=m_1M_sL_* * (\lambda_{i*}-\lambda_{j*})/(kT\xi_*)$; $\alpha=(kT/(m_1M_s))^3 * (\xi_*/L_*)^4\beta$; $f(\xi):=\ln(\operatorname{sh}\xi/\xi)-\xi L(\xi)$. The parameters τ_λ, τ_ν, D_λ, D_ν and $\beta(<0)$ are considered permanent. In addition, assume that the condition of paramagnetic media: $3kT\nu>m_1M_s\lambda$. In this case the equation of electrodynamics in (1.6) and boundary conditions for the magnetic field are satisfied automatically.

Homogeneous equilibrium state of liquid $\lambda=\lambda_h=\text{const}$, $\nu=\nu_h=\text{const}$ satisfy the equations:

$$Q^{(\lambda)}(\lambda,\nu;H)=0,\quad Q^{(\nu)}(\lambda,\nu;H)=0. \tag{2.2}$$

With $\lambda_{1*}\neq\lambda_{2*}$ the functions $\lambda(M)$, $\nu(M)$ defined by these equations, ambiguous, fluid has a smooth self-intersecting curve of magnetization, which consists of nine branches of the uniqueness of the function $M(H)$, and in different ranges of the magnetic field is from one to nine of equilibrium states [14]. The stability of these states is ensured by

$$Q_\lambda^{(\lambda)}<0,\quad Q_\nu^{(\nu)}<0,\quad \Delta:=Q_\lambda^{(\lambda)}Q_\nu^{(\nu)}-Q_\lambda^{(\nu)}Q_\nu^{(\lambda)}>0. \tag{2.3}$$

Analysis of these conditions shows that if the inequality

$$-\frac{\beta\langle\lambda\rangle^2}{6}>\frac{m_1M_sL_*^2}{kT\nu_{2*}/L'(\xi_*)-m_1M_s\lambda_{3*}} \tag{2.4}$$

two branches of the magnetization curve contains asymptotically stable homogeneous equilibrium state.

The nature of the system (2.1) is determined by the type of local coupling curve, which is defined on the plane (ν,λ) with the equation $Q^{(\nu)}(\lambda,\nu;H)=0$ at $H=\text{const}$. In [17] have shown that there exists a range of values of the magnetic field in which the local coupling curve for a magnetizable fluid has V – or И – shaped, i.e. the system under consideration is a V - or И – system [21], respectively. Another important characteristic is the curve equation of state, which given on the same plane with the equation $Q^{(\lambda)}(\lambda,\nu;H)=0$ at $H=\text{const}$. For magnetizable fluid in a specified range of values H it has S – shaped.

For the practice of interest to the case K – the system [37], for which $D_\nu<D_\lambda$. In K – a system parameter λ varies in space much slower than the parameter ν, and ν is a fast variable, and λ – the slow variable.

In this paper we consider stationary heterogeneous solutions of (3.1) type of static autosolitons and periodic strata.

2.2. The layered and strip structures. Let magnetizable fluid is placed between two parallel infinite nonmagnetic plates in a magnetic field parallel to the plates. The beginning of a rectangular Cartesian coordinate system O chosen in the middle of the liquid layer, the axis Ox is directed perpendicular to the layer, and the axis Oy – along the magnetic field. Consider the one-dimensional stationary solutions of (2.1) which satisfy the Neumann boundary conditions.

Strata in the small-size systems [18]. In the fluid layer for which $D_\nu < L^2 < D_\lambda$ (small system), the effective magnetic field parameter takes almost the same value for all liquid particles: $\lambda = \lambda_c = \text{const}$ and the function $\nu(x)$ is a solution

$$\nu'' + Q^{(\nu)}(\lambda_c, \nu(x); H) = 0,\ \nu'\left(-\frac{L}{2}\right) = \nu'\left(\frac{L}{2}\right) = 0,\ \int_{-L/2}^{L/2} Q^{(\lambda)}(\lambda_c, \nu(z); H)dz = 0.$$

Here the prime denotes the derivative with respect to the dimensionless variable $z = x/\sqrt{D_\nu}$, $l := L/\sqrt{D_\nu}$, L – the distance between the plates. The last relation is obtained by integrating the first equation (2.1) over the thickness of the liquid layer.

The phase portrait of dynamical system, which reduces the first equation is determined by the quantity λ_c. When $\lambda_- < \lambda_c < \lambda_+$, where λ_-, λ_+ – the point of maximum and minimum of the local coupling curve, respectively, the system has three fixed points (denoted by ν_1, ν_2, ν_3; $\nu_1 < \nu_2 < \nu_3$) saddle points ν_1, ν_3, and center ν_2. When $\int_{\nu_1}^{\nu_3} Q^{(\nu)}(\lambda_s, \nu; H)d\nu = 0$ on the phase plane of a cell is bounded by two separatrices, one of which comes from the saddle ν_1 and is in a saddle ν_3, while the other goes out of the saddle ν_3 and is in the saddle ν_1. Close to the boundary of the cell closed trajectory corresponds to a wide stratum at the center of the liquid layer. Stratum contains sections smooth and sharp variation of the parameter ν: in the plane (λ, ν) smooth changes occur in small neighborhoods of points (λ_s, ν_1), (λ_s, ν_3) and abrupt changes – along the segment $\nu_1 < \nu < \nu_3$ of the line $\lambda = \lambda_s$. For the hot (cold) strata the average number of particles in aggregates in the middle of the segment $-l/2 < z < l/2$ more (less) than near the boundary.

Except for a single hot or cold strata the problem has a lot of periodic solutions in the form of a sequence of identical strata of the period $l_p = l/N$ (N – a natural number).

Wide static autosolitons [19]. In case $D_\nu < D_\lambda < L^2$ (large system), change the parameter of the effective magnetic field λ in the fluid is comparable to the change of the parameter ν, and the equation describing the distribution λ and ν in liquid have the form

$$\lambda'' + Q^{(\lambda)}(\lambda, \nu; H) = 0 \qquad \varepsilon^2\nu'' + Q^{(\nu)}(\lambda, \nu; H) = 0, \tag{2.5}$$

where the prime denotes the derivative with respect to the dimensionless variable $z = x / \sqrt{D_\lambda}$, $\varepsilon = \sqrt{D_\nu / D_\lambda} < 1$.

Equations (2.5) can be represented as a dynamic system of fourth order:

$$\varepsilon X_i' = f_i,\ i = 1,2, \qquad X_i' = f_i,\ i = 3,4, \tag{2.6}$$

where $X_1 = \nu$, $X_2 = \varepsilon\nu'$, $X_3 = \lambda$, $X_4 = \lambda'$, $f_1 = X_2$, $f_2 = -Q^{(\nu)}(X_3, X_1; H)$, $f_3 = X_4$, $f_4 = -Q^{(\lambda)}(X_3, X_1; H)$.

The first two equations (2.6) contain a small parameter at the derivative and the system is singularly perturbed [39]. In the case where the function determining the dependence $Q^{(\nu)}$ of ν at the fixed λ and H has three roots, there may be a solution describing the transition ν from root to root, or a decision with an internal transition layer. This decision determines the static autosoliton [37]. Static autosoliton is symmetric with respect to the point $z = 0$, for $z \to \pm\infty$ is seeking to homogeneous equilibrium state $\lambda = \lambda_h$, $\nu = \nu_h$, and at some value $z = z_0$ (transition point) fast variable abruptly from one root to another root.

Width of the autosoliton $l_s = 2z_0$ is given by

$$l_s = \sqrt{2}\int_{\lambda_s}^{\lambda_m} d\lambda \left[\sqrt{\int_{\lambda}^{\lambda_m} Q^{(\lambda)}(\lambda, \nu_I(\lambda); H)\, d\lambda}\right]^{-1/2}.$$

Periodic structures. The problem also allows for periodic solutions in the form of the sequence the same strata.

Changing the parameter of the effective magnetic field in autosoliton is described by homoclinic separatrix. Closed orbits which are located inside the separatrix, determine the dependence $\lambda(z)$ in the periodic strata. The width of the stratum is given by

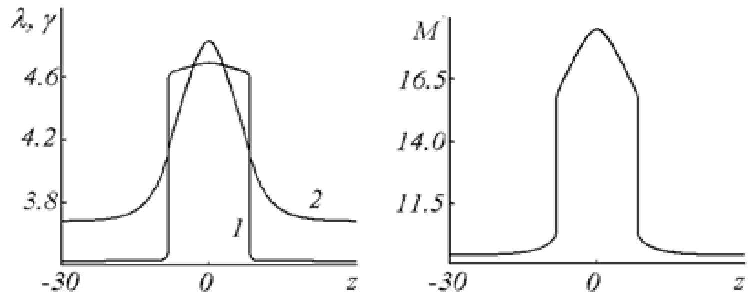
$$l_s = \sqrt{2}\int_{\lambda_s}^{\lambda_n} d\lambda \left[\int_{\lambda}^{\lambda_n} Q^{(\lambda)}(\lambda, \nu(\lambda); H)\, d\lambda\right]^{-1/2},$$

and the period

$$l_p = l_s + \sqrt{2}\int_{\lambda_t}^{\lambda_s} d\lambda \left[\int_{\lambda}^{\lambda_t} Q^{(\lambda)}(\lambda, \nu_{III}(\lambda); H)\, d\lambda\right]^{-1/2}.$$

Values l_s and l_p depend on the magnitude of λ_t which determines the intersection point of the trajectory with the axis of X–axis, and field strength. When $\lambda_t \to \lambda_h$ the period tends to infinity, and the shape of the stratum tends to autosoliton form. If $\lambda_t \to \lambda_s$, then $\lambda(z) \to \lambda_s = \text{const}$, $\nu(0) \to \nu_1$, $\nu(l_p/2) \to \nu_3$, and the width of the stratum and period tend to zero.

The obtained solutions describe the formation of one or more equally spaced sub-layers, in which aggregates of ferroparticles are concentrated in a magnetizable fluid layer (layered structure). These solutions simulate the formation of stationary structures in fluid in the case when the axis Ox is parallel to the layer too. They are implemented as directed along the magnetic field separated bands of a more aggregated continuum (strip structures). Periodic strata are observed experimentally in films of magnetic fluid [1–4]. Comparing the calculated values of the parameters of periodic strata with experiment, we can find estimates for the parameters $\tau_\lambda, \tau_\nu, D_\lambda, D_\nu$ of the model fluid with transformed microstructures used in this paper: $\tau_\nu \sim 10\,\text{s}$, $\tau_\lambda \sim 7200\,\text{s}$, $D_\lambda \approx 2\,\text{mkm}^2$, $D_\nu < 1\,\text{mkm}^2$ [19].



a b

Fig.2. Static autosoliton in magnetizable fluid (*a* – distribution of parameters λ and $\gamma = \nu^{-1}$; *b* – distribution of the magnetization M)

Contrast structures (spike autosolitons) [16]. In the range of values of the magnetic field strength at which the local coupling curve is a V – shaped, conditions are satisfied:

$$Q^{(\lambda)}(\lambda_h, \nu_h; H) = 0, \quad Q^{(\nu)}(\lambda_h, \nu_h; H) = 0, \quad Q^{(\nu)}(\lambda_h, \nu_n; H) = 0.$$

$$Q_\nu^{(\nu)}\ (\lambda_h, \nu_h; H) < 0, \quad Q_\nu^{(\nu)}\ (\lambda_h, \nu_n; H) > 0, \quad \int_{\nu_h}^{\nu_s} Q^{(\nu)}(\lambda_h, \nu; H) d\nu = 0$$

$$\int_{\nu_h}^{\nu_s} \frac{Q^{(\lambda)}(\lambda_h, \nu; H)}{\sqrt{Q^{(\nu)}(\lambda_h, \nu; H)}} d\nu \neq 0, \qquad \Delta(\lambda_h, \nu_h; H) > 0.$$

In this case, the parameters λ_h, ν_h define an asymptotically stable homogeneous equilibrium state of a fluid, for a given H function $Q^{(\nu)}(\lambda_h, \nu; H)$

has two roots $\nu = \nu_h, \nu = \nu_n$, and the system (2.1) has a contrast structure or spike autosoliton solution.

This solution has a point of "splash" of the third type [39] $x_* = 0$ and is characterized by a sharp change in the average number of particles in aggregates in the $\sqrt{D_\nu}$ − neighborhood of x_*, but a weak variation of the parameter of the effective magnetic field $(\lambda \approx \lambda_h)$ in the entire volume of the fluid.

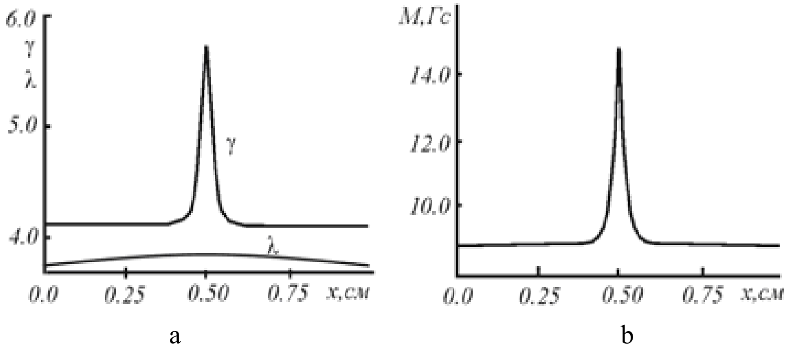


Fig.3. Contrast structure (spike autosoliton) in the magnetizable fluid (*a* – distribution of the parameters λ and $\gamma = \nu^{-1}$; *b* – distribution of the magnetization M).

The leading term of ν in the small parameter ε has the form

$$\nu(x) \sim \nu_h + \Pi\nu(\tau), \qquad \nu(x) = \nu(-x), \qquad 0 \le x \le \frac{L}{2},$$

where $\tau = x/\varepsilon$.

The function $\Pi\nu(\tau)$ is defined as the solution of the problem

$$\frac{d\Pi\nu}{d\tau} = \mathrm{sign}(\nu_h - \nu_s)\Phi(|\Pi\nu|), \qquad \Pi\nu(0) = \nu_s - \nu_h$$

and has the usual for boundary function exponential estimate

$$|\Pi\nu(\tau)| \le C\exp(-p\tau), \qquad C > 0, \qquad p > 0.$$

Here

$$\Phi(|x|) = \left[-\frac{2}{D_\lambda}\int_0^x Q^{(\nu)}(\lambda_h, \nu_h + t; H)dt \right]^{1/2},$$

and the coefficient p satisfies

$$p \le \left(-\frac{1}{D_\lambda} Q_\nu^{(\nu)}(\lambda_h, \nu_h; H) \right)^{1/2}.$$

The asymptotic approximation for λ with the accuracy of the order ε is

$$\lambda(x)=\lambda_h+\varepsilon\,\frac{\eta}{2\kappa\,\mathrm{sh}(\kappa L/2)}\begin{cases}\mathrm{ch}\left[\,\kappa(x+L/2)\right], & -L/2\le x<0\\ \mathrm{ch}\left[\,\kappa(x-L/2)\right], & 0\le x\le L/2\end{cases},$$

where

$$\kappa=\sqrt{-\frac{\Delta(\lambda_h,\nu_h;H)}{D_\lambda Q_\nu^{(\nu)}(\lambda_h,\nu_h;H)}},$$

$$\eta=-\sqrt{\frac{2}{D_\lambda}}\int\limits_{\nu_h}^{\nu_s}Q^{(\lambda)}(\lambda_h,\nu_h;H)\Big/\left[\int\limits_{\nu}^{\nu_h}Q^{(\nu)}(\lambda_h,\xi;H)d\xi\right]^{-1/2}d\nu\,.$$

In different ranges of H solutions exist with a positive and a negative "splash" value $1/\nu_s-1/\nu_h$, while the average number of particles in the aggregates takes values $\sim 4-6$.

Functions $\lambda(x)$, $\nu(x)$ are even with respect to $x_*=0$, possible to continue these functions in the axis Ox and construct of solutions with a few points of "splash", arranged uniformly on the interval $[-L/2, L/2]$. These functions define the layered structures in a magnetizable fluid.

If the axis Ox is parallel to the layer, the solution describes the oriented along the field needle configuration of the ferroparticles (strip structures), they are observed experimentally in films [1,3–5] and in the volume of a magnetizable fluid [34].

3. Conclusions. The mean-field approximation in the hydrodynamics of magnetizable media and its relationship with the classical theory of magnetization are analyzed in detail. Equilibrium thermodynamic models for structuring e of magnetic fluids are proposed and a closed boundary-value problem of structuring processes in the framework of the phenomenological Landau theory of phase transitions is formulated. Results of the study of spatial patterns of ferroparticles formed in a volume of magnetic fluid under the influence of a constant homogeneous magnetic field are presented. The equations describing the evolution of the magnetic state of the fluid, determined, depending on the field, KV– or KИ–system where the average number of particles in aggregates is a fast variable and the parameter of the effective magnetic field – a slow variable. Using the theory of autosolitons stationary solutions of the equations of static autosolitons and periodic strata are found. The resulting solutions are realized in the form of parallel layers (layered structure) or stripes (stripe structure), in which aggregates of ferroparticles are concentrated. The strip structures are observed experimentally in films of magnetic fluid. Comparison of the calculated values of the geometric characteristics of the periodic strata and experiment allows us to find estimates for the parameters used in the work model of the fluid with transformed microstructure.

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