STATISTICAL TWO-LEVEL MODEL OF THE PRODUCTION PROCESS

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Statistical Physics essentially widens the field of its application, penetrates into such closely-related fields of knowledge, as Chemistry, Biology, Meteorology. Statistical Physics is one of the main instruments for description of system's self-organization in such comparatively new field of knowledge, as 'Synergetic Economics'. Representation of any industrial works with mass production output as the system with numerous amount of elements (base products), with stochastic character, being in the manufacturing process, makes possible the application of Statistical Physics for the system's methods description. The regularities, which are characteristics of the states of equilibrium in the systems of economical exchange, are sizeably analogous to them, which are in physical (thermodynamic) systems. They proved to be so considerable and useful, that they were proclaimed for thermodynamic systems and systems of economical exchange as some general principles, they are: Le–Shatelue–Samuelson, Carnot–Khix, etc. [1]. The term 'synergetics' introduced by Hacken has gained quite broad admission as a common name of the interdisciplinary field dealing with study of complicated processes and systems' self-organization. Synergetics, as the science dealing with self organization, includes the community of interests and mathematical methods of relative non-linear phenomena in large systems conduct research. Obviously, research of relationships, able to exist between deterministic dynamics and probability processes of separate elements conduct, is important for formulation of large systems conduct. It is well-known, that the stochastic processes allow formulating the models of large systems, able to describe the irreversible evolution of the system and assume the Lyapunov functional or H–function, conduct. Thus, the problem is in the conversion from the deterministic dynamics to the probability process. The procedure of the main kinetic equation obtaining from the separate elements dynamics usually begins with the description shortening or cells enlargement. On the basis of these principles of large systems description of contemporary mass production can be represented as the stochastic process, during which the manufacturing system changes one state into another [2, p.178].

The aim is to study and develop theoretical foundations and conceptual positions of statistical modeling of production and technical systems technological processes in order to improve planning and management processes of products manufacturing.

It has defined the research problem. 1. Conceptual framework of the statistical theory of modeling the process control systems was developed. 2.Substantiate the necessity of statistical approach to the construction of mathematical models of existing processes. 3. Develop a general two-level way of modeling a manufacturing process that takes into account the stochastic nature of the process equipment influence on the objects of labor and collective processes of subjects of labor interaction among themselves.

Object of study: manufacturing process of production and technical systems.

1. Mathematical model. The state of the system can be defined as the state of the total amount N of the manufacturing system's base products. The term 'base product' (or the object of labour) one should understand as the element of manufacturing system, which is subjected to the transfer of the human labour, raw materials and labour implements cost in accordance with its motion through the operational chain of manufacturing routes. During the motion the transformation of initial raw materials into final product by means of purposeful influence of publically useful labour. Let us describe the base product state as microscopic rates $\left(S_j,\mu_j\right)$, here S_j (hrn) and $\mu_j = \lim_{\Delta t \to 0} \frac{\Delta S_j}{\Delta t} \left(\mathrm{hrn}/\mathrm{hrs} \right)$, which constitute conformably the sum of all expenses at time unit, transferred by manufacturing system to the j-th base product, $0 < j \le N$. The conduct of the main of the base product follows definite laws in accordance with the manufacturing process, prescribed in industrial works, the manufacturing plan, the human activities and equipment, and can be defined as the engineering–production function $f_j(t,S)$. The state of the system at any time moment is defined, if the microscopic rates $\left(S_1,\mu_1;........S_N,\mu_N\right)$ are defined too, and at any other time moment it can be obtained with the help of the base products' state equations:

$$\frac{dS_j}{dt} = \mu_j, \qquad \frac{d\mu_j}{dt} = f_j(t,S) . \tag{1}$$

However, if the amount of base products N is much larger than one, it is almost impossible to solve the system (1) of $2 \cdot N$ –equations. This amplification requires the transfer from the microscopic description of manufacturing system to the macroscopic one with the elements of probability nature. The main difficulty of such description is in extraction of the features of the base products' microscopic states, which can be commensurable on the level of industrial works' state. Instead of the state of manufacturing system with microscopic rates $\left(S_1,\mu_1;........S_N,\mu_N\right)$ consideration, let us input according to this the normalised discrete function of the amount of base products N distribution in the phase space (S,μ). Each point of this space will be given as the base product state. The fact, that, if the values of N are large, this function will be comfortably approximated by the continuous function of base products $\chi(t,S,\mu)$ distribution, is worth waiting. If the manufacturing system consists of several types of base products, the distribution function for each type of base product will be required for the system's description. Let us divide the phase space into such number of cells, during which the cell's dimensions $\Delta\Omega = \Delta S \cdot \Delta\mu$ are much smaller than the manufacturing system characteristic dimensions and, at the same time, include large number of base products. Instead of the precise values of the base product microscopic rates fixed, let us approximately characterise the state of the manufacturing system by the number of base products in each cell $\Delta\Omega$. If the cell's dimensions are small enough, the approximate description will include almost the same detailed information as the precise one. Thus, it gets necessary to consider the limiting case, when the cell's dimensions tend to zero, together with the main limit, when $N \to \infty$. On the strength of the rate $\chi(t,S,\mu)\cdot d\Omega$ looking as the amount of base products in the infinitely small cell $\Delta\Omega$ of the phase state (S,μ), we are able on the basis of the phase coordinate S and the phase rate μ of base product change in time to see the change of the function $\chi(t,S,\mu)$ [3]:

$$\frac{\partial\chi}{\partial t} + \frac{\partial\chi}{\partial S}\cdot \mu + \frac{\partial\chi}{\partial\mu}\cdot f(t,S) = J(t,S,\mu) \tag{2}$$

The rate of expenses change μ of the base product and the function f(t,S) can be obtained by means of the system of the base product state equations (1):

$$\frac{dS}{dt} = \mu; \qquad \frac{d\mu}{dt} = f(t,S) \tag{3}$$

and the generative function $J(t,S,\mu)$ can be defined by means of the equipment density lengthwise the technological change and its technical features.

Movement of items of work in technological route is described by the coordinates (S,μ). The first coordinate describes the change in value of the subject of labor in the transition from one process step to another, the second – to change the intensity of the transfer of technology resources on the subject of work in interaction with technological equipment. Position the object of labor is given a point in space with coordinates (S,μ). Position N items of work which are in the process, represented by N points. The subject of work, passing technological treatment from one state to another, describes the trajectory (1). The distribution function of the states of the objects of labor is introduced so that the product $\chi(t,S,\mu)\cdot dS\cdot d\mu$ gives the number of items of work that are in an element near the point with coordinates S and μ. If all space is partitioned into elements that can be written approximately $\sum_i\sum_j \chi(t,S_i,\mu_j)\cdot\Delta\mu_j\cdot\Delta S_i \approx \int_0^{Sd} dS\cdot\int_0^{\infty}\chi(t,S,\mu)\cdot d\mu$.

The summation $\sum_i\sum_j \chi(t,S_i,\mu_j)\cdot\Delta\mu_j\cdot\Delta S_i$ of all unit cells gives the total number of items of work N, located in the technological processing $\sum_i \sum_j \chi\left(t, S_i, \mu_j\right) \cdot \Delta\mu_j \cdot \Delta S_i = N$. Function $\chi(t, S, \mu)$ is integrable (Riemann).

The function $J(t, S, \mu)$, when $t \to \infty$, strives to break the base product initial distribution down the rates of expenses change to the state with the equilibrium function $\chi(t, S, \mu)$ as the normalized one

$$\int_0^{Sd} dS \cdot \int_0^{\infty} d\mu \cdot \chi\left(t, S, \mu\right) = N \tag{4}$$

S_d –product costs. The engineering production function f(t,S) can be defined from the industrial works documentation, that are the tables of the raw materials expenditures rates, normalized raw materials prices for the manufacturing operations realization by a worker. Having expressed by means of money from the industrial works documentation the cost of raw materials expenditures, which are required during the manufacturing operation, the time and the price of the manufacturing operation realization, it's possible to obtain the relationship for the expenses change rate $\mu_\kappa = \mu(t_\kappa)$ while moving of the base product lengthwise the technological chain $f(t, S) = \lim_{\Delta t_k \to 0} \frac{\Delta\mu_k}{\Delta t_k}$ in the form of table. According to its own meaning the engineering production function looks like an analogue to force, moving the base product lengthwise the technological chain of manufacturing process. In the conditions of such motion the base product is influenced by the labour implements (equipment). Thus, the increase of expenses, transferred to the base product while it is moving lengthwise the technological chain of manufacturing process, has place. Equipment influences the base product, changing it both qualitatively and quantitatively. But we can only speak about the probability of the base product being in that or another state after the influence on it by means of the technological equipment. The process of influence on the base product by means of technological equipment let us designate as $\left[\mu \to \tilde{\mu}\right]$, here μ and $\tilde{\mu}$ are the rates of expenses change of the base product before and after influence, correspondingly. But the total quantity of the base products, being in the volume unit of the phase space and being influenced by the technological equipment at a time unit, can be put down as the product of the base products' flow $\chi(t, S, \mu) \cdot \mu$ and the probability of being influenced for each of them $\left[\mu \to \tilde{\mu}\right]$ in any small element dΩ of the phase space (S,μ). As for the probability of being influenced $\left[\mu \to \tilde{\mu}\right]$, one can, at least, confirm that the probability of being influenced is proportional to the density of the equipment disposition λ equipment lengthwise the technological chain. Thus, the number of base products, having been influenced by the technological equipment at a time unit and having the values in the range $(\tilde{\mu}; \tilde{\mu} + d\tilde{\mu})$, we can put down as

$$\psi\left[\mu\to\tilde{\mu}\right]\cdot\lambda_{\text{equipment}}\cdot\mu\cdot\chi(\mathrm{t},\mathrm{S},\mu)\cdot\mathrm{d}\tilde{\mu}\cdot\mathrm{dS}\cdot\mathrm{d}\mu \tag{5}$$

here $\psi\left[\mu\to\tilde{\mu}\right]$ is the function, which is defined by the manufacturing characteristics of the technological equipment functioning. Some properties of this function one can obtain using general ideas, if we imagine, that the total probability of the transfer into any state equals to one:

$$\int_0^\infty \psi\left[\mu\to\tilde{\mu}\right]\cdot\mathrm{d}\tilde{\mu}=1 \tag{6}$$

Together with this fact the base products come up in the volume element $\mathrm{dS}\cdot\mathrm{d}\mu$ from the volume $\mathrm{dS}\cdot\mathrm{d}\tilde{\mu}$ by means of the inverse transfer $\psi\left[\tilde{\mu}\to\mu\right]$ with the following quantity:

$$\psi\left[\tilde{\mu}\to\mu\right]\cdot\lambda_{\text{equipment}}\cdot\tilde{\mu}\cdot\chi(\mathrm{t},\mathrm{S},\tilde{\mu})\cdot\mathrm{d}\tilde{\mu}\cdot\mathrm{dS}\cdot\mathrm{d}\mu \tag{7}$$

and the total number of base products in the volume element dΩ changes its rate at a time unit:

$$\mathrm{d}\Omega\cdot\mathrm{J}=\mathrm{d}\Omega\cdot\lambda_{\text{equipment}}\cdot\int_0^\infty\left\{\psi\left[\tilde{\mu}\to\mu\right]\cdot\tilde{\mu}\cdot\chi(\mathrm{t},\mathrm{S},\tilde{\mu})-\psi\left[\mu\to\tilde{\mu}\right]\cdot\mu\cdot\chi(\mathrm{t},\mathrm{S},\mu)\right\}\mathrm{d}\tilde{\mu} \tag{8}$$

Taking into consideration the normalizing property (6) of the function $\psi\left[\mu\to\tilde{\mu}\right]$, the equation (2) will look like:

$$\frac{\partial\chi}{\partial \mathrm{t}}+\frac{\partial\chi}{\partial \mathrm{S}}\cdot\mu+\frac{\partial\chi}{\partial\mu}\cdot\mathrm{f}(\mathrm{t},\mathrm{S})=\lambda_{\text{equipment}}\cdot\left\{\int_0^\infty\left[\psi\left[\tilde{\mu}\to\mu\right]\cdot\tilde{\mu}\cdot\chi(\mathrm{t},\mathrm{S},\tilde{\mu})\right]\cdot\mathrm{d}\tilde{\mu}-\mu\cdot\chi\right\} \tag{9}$$

In the most cases, interesting from the viewpoint of practice, the function $\psi\left[\mu\to\tilde{\mu}\right]$ doesn't depend on the base product state before the influence of the technological equipment, that leads to simplification of integral–differential equation (9):

$$\frac{\partial\chi}{\partial \mathrm{t}}+\frac{\partial\chi}{\partial \mathrm{S}}\cdot\mu+\frac{\partial\chi}{\partial\mu}\cdot\mathrm{f}(\mathrm{t},\mathrm{S})=\lambda_{\text{equipment}}\cdot\left\{\psi\left[\tilde{\mu}\to\mu\right]\cdot\left[\chi\right]_1-\mu\cdot\chi\right\} \tag{10}$$

The zero $\int_0^\infty \mathrm{d}\mu\cdot\chi\left(\mathrm{t},\mathrm{S},\mu\right)=\left[\chi\right]_0$ and the first moments $\int_0^\infty \mathrm{d}\mu\cdot\mu\cdot\chi\left(\mathrm{t},\mathrm{S},\mu\right)=\left[\chi\right]_1=\langle\mu\rangle\cdot\left[\chi\right]_0$ of the distribution function have simple manufacturing interpretation: surpluses of the base products and the pace of their motion lengthwise the technological chain.

2. Main results. With the help of the distribution function moments one can obtain the system of equations for the manufacturing system macroscopic rates description. Multiplying the equation (10) correspondingly by $1, \mu, \mu^2$ and integrating it on the whole range of μ, we'll obtain the equations of balances [4]:

$$\frac{\partial\left[\chi\right]_0}{\partial \mathrm{t}}+\frac{\partial\left(\langle\mu\rangle\cdot\left[\chi\right]_0\right)}{\partial \mathrm{S}}=\int_0^\infty \mathrm{d}\mu\cdot\mathrm{J}, \tag{11}$$

$$\frac{\partial\left(\langle\mu\rangle\cdot[\chi]_0\right)}{\partial t}+\frac{\partial\left(\langle\mu\rangle^2\cdot[\chi]_0\right)}{\partial S}=-\frac{\partial P}{\partial S}+f(t,S)\cdot[\chi]_0+\int_0^\infty d\mu\cdot\mu\cdot J\,,$$

$$\frac{\partial}{\partial t}\left\{\frac{\langle\mu\rangle^2\cdot[\chi]_0}{2}+\frac{P}{2}\right\}+\frac{\partial}{\partial S}\left(\langle\mu\rangle\cdot\left\{\frac{\langle\mu\rangle^2\cdot[\chi]_0}{2}+\frac{3\cdot P}{2}\right\}+\Theta\right)=f(t,S)\cdot[\chi]_1+\int_0^\infty d\mu\cdot\frac{\mu^2}{2}\cdot J$$

in which $\int_0^\infty d\mu\cdot\left(\mu-\langle\mu\rangle\right)^2\cdot\chi=P(t,S)$; $\int_0^\infty d\mu\cdot\left(\mu-\langle\mu\rangle\right)\cdot\frac{\left(\mu-\langle\mu\rangle\right)^2}{2}\cdot\chi=\Theta(t,S)$.

The balance equations (11), which we consider as the surpluses equations, the equations of pace and dispersion of the base products lengthwise the technological chain, are not closed. The possibility of obtaining of the closed system of equations is based on the properties of the function $\psi\left[\mu\to\tilde{\mu}\right]$ and presence of the small parameter $Kv=\frac{l_{free}}{\xi}<<1$ [3], which can be considered as the relationship between the length of the base products free motion /free lengthwise the technological chain among the equipment unit and characteristic dimension of the technological chain. In the zero approximation according to the small parameter $Kv<<1$ we have

$$J=\sum_{m=0}^{\infty}\left(Kv\right)^m\cdot J_m;\;\; J_0=\lambda_{equipment}\cdot\left\{\psi\left[\tilde{\mu}\to\mu\right]\cdot[\chi]_1-\mu\cdot\chi\right\}=0 \qquad (12)$$

and from the equation of balances (11) the closed system of equations for the manufacturing system description is obtained:

$$\frac{\partial[\chi]_0}{\partial t}+\frac{\partial\left([\chi]_0\cdot\langle\mu\rangle\right)}{\partial S}=0;\;\; \frac{\partial\langle\mu\rangle}{\partial t}+\langle\mu\rangle\cdot\frac{\partial\langle\mu\rangle}{\partial S}=-\frac{1}{[\chi]_0}\cdot\frac{\partial P(t,S)}{\partial S}+f(t,S);$$

$$\frac{1}{2}\cdot\frac{\partial P(t,S)}{\partial t}+P\cdot\frac{\partial\langle\mu\rangle}{\partial S}+\frac{\partial\Theta_\Psi}{\partial S}=0\,, \qquad (13)$$

here $\Theta_\Psi=\langle\mu\rangle\cdot\left\{\frac{[\chi]_0\cdot\sigma_\psi^{\,2}}{2}+\frac{[\chi]_0\cdot\left(\mu_\psi-\langle\mu\rangle\right)^2}{2}\right\}$, $\Theta(t,S)=\Theta_\Psi(t,S)-\frac{\langle\mu\rangle}{2}\cdot P$,

and μ_ψ and σ_ψ^2 are defined as $\int_0^\infty\mu\cdot\psi\left[\mu\to\tilde{\mu}\right]\cdot d\tilde{\mu}=\mu_\psi$;

$\int_0^\infty\left(\mu-\mu_\psi\right)^2\cdot\psi\left[\mu\to\tilde{\mu}\right]\cdot d\tilde{\mu}=\sigma_\psi^{\,2}$ and are usually given by the manufacturing characteristics of equipment.

Let us suppose that the pace of the base products $[\chi]_1(t,S)$ lengthwise the technological chain is given then as a particular case from the system of equations (13) we obtain Forrester's equation [5, p.170], which is well-known in cybernetic economics. The closed system of balances equations is considerably simplified if the manufacturing process is permanent. The permanent (or established) manufacturing process has place if in each point of the technological chain the macroscopic rates $[\chi]_0$, $\langle\mu\rangle$, P remain unchangeable in time

$$\frac{\partial[\chi]_0}{\partial t}=0, \qquad \frac{\partial\langle\mu\rangle}{\partial t}=0, \qquad \frac{\partial P}{\partial t}=0. \tag{14}$$

In this case the system of balances equations (13) looks like

$$\frac{\partial\left([\chi]_0\cdot\langle\mu\rangle\right)}{\partial S}=0;\ \langle\mu\rangle\cdot\frac{\partial\langle\mu\rangle}{\partial S}=-\frac{1}{[\chi]_0}\cdot\frac{\partial P(S)}{\partial S}+f(S);\ P\cdot\frac{\partial\langle\mu\rangle}{\partial S}+\frac{\partial\Theta_\Psi}{\partial S}=0 \tag{15}$$

$$\text{or } [\chi]_1=\text{const};\ [\chi]_0\cdot\langle\mu\rangle^2+P(S)+F(S)=\text{const};\ P\cdot\frac{\partial\langle\mu\rangle}{\partial S}+\frac{\partial\Theta_\Psi}{\partial S}=0. \tag{16}$$

The integral engineering–production function F(S) is defined as $f(S)\cdot[\chi]_0=-\frac{\partial F(S)}{\partial S}$. It is a well-known fact, that small disturbing factors' influence upon the manufacturing–sale system is not the same for different processes. This influence upon some technological processes is insignificant as the disturbed state differs a little from the undisturbed one. And vice versa, the influence of distortions upon other technological processes is quite significant and doesn't depend on how small the disturbing impacts are. As the disturbing factors inevitably exist, it gets obvious, that the task of the manufacturing process' stability appears to have a very important theoretical and practical meaning. Let us consider the investigation of manufacturing process' stability in terms of macroparameters of manufacturing process with the mass production output. System's macroparameters are the base product surpluses $[\chi]_0$ between the technological operations lengthwise the manufacturing process' technological chain and the base products' pace motion [$[\chi]_1$ from one operation to another, which represents the zero $[\chi]_0$ and the first $[\chi]_1$ moments of the distribution function (10). While saying 'disturbing factors', let us consider the impact, which was not taken into account in the description of manufacturing process, because of impact's smallness in comparison with the main factors, that influence the manufacture and the production output. This factors may act both instantly (that leads to the insignificant modification of the manufacturing system' original state) and constantly (that means, that the set manufacturing process' equations differ from real onto some small correction numbers, that were not taken into consideration in the equations of manufacturing process).

The closed system of equations (13) for the macroparameters of the manufacturing system in zero approximation on the small parameter $Kv=(l_{св}/\xi)<<1$, can be simplified to the following appearance:

$$\frac{\partial[\chi]_0}{\partial t}+\frac{\partial[\chi]_1}{\partial S}=0 \tag{17}$$

$$\frac{\partial[\chi]_1}{\partial t}+\frac{\partial[\chi]_2}{\partial S}=f(t,S)\cdot[\chi]_0, \qquad [\chi]_2=[\chi]_1\cdot\left(\frac{[\chi]_{1\psi}}{[\chi]_0}\right) \tag{18}$$

where $[\chi]_2$ is the second moment of the base product distribution function $\chi(t,S,\mu)$ over the rates of inputs change μ, and the macroscopic value $[\chi]_{1\psi}$ is defined by the technological characteristics of the equipment [3].

Let's suppose, that, in order to describe the manufacturing process, the undisturbed solution (19) corresponds to the system of equations (17)

$$[\chi]_0^*=[\chi]_0^*(t,S), \qquad [\chi]_1^*=[\chi]_1^*(t,S) \tag{19}$$

Solution (19) of the system of equations (2) corresponds to the planned indices of the manufacturing process. The manufacturing plan expresses the balance between the base products' motion pace lengthwise the technological chain and the required monthly production output. Let us concede, that the values, under observation, technological surpluses $[\chi]_0$ and base products' motion pace lengthwise the technological chain $[\chi]_1$ are impacted by the small fortuitous distortions $[y]_0$, $[y]_1$ correspondingly to their undisturbed state (19):

$$[y]_0=[\chi]_0-[\chi]_0^*; \qquad [y]_0=[\chi]_0-[\chi]_0^*; \tag{20}$$

Let's linearize the manufacturing macroscopic system (14) relatively to the small distortions (20) about the undisturbed state (19):

$$\frac{\partial[y]_0}{\partial t}+\frac{\partial[y]_1}{\partial S}=0, \tag{21}$$

$$\frac{\partial[y]_1}{\partial t}+\frac{\partial[y]_1}{\partial S}\cdot B_{\left(\partial[y]_1/\partial S\right)}+[y]_1\cdot B_{([y]_1)}+\frac{\partial[y]_0}{\partial S}\cdot B_{\left(\partial[y]_0/\partial S\right)}+[y]_0\cdot B_{([y]_0)}=0$$

where the input coefficients are:

$$B_{\left(\partial[y]_1/\partial S\right)}=\frac{[\chi]_{1\psi}}{[\chi]_0^*}, \quad B_{([y]_1)}=\frac{\partial}{\partial S}B_{\left(\partial[y]_1/\partial S\right)}-\left.\frac{\partial\left(f(t,S)\cdot[\chi]_0\right)}{\partial[\chi]_1}\right|_{[\chi]_0=[\chi]_0^*,[\chi]_1=[\chi]_1^*} \tag{22}$$

$$B_{\left(\partial[y]_0/\partial S\right)}=-\frac{[\chi]_1^*\cdot[\chi]_{1\psi}^*}{\left([\chi]_0^*\right)^2}, \quad B_{([y]_0)}=-\frac{\partial}{\partial S}B_{\left(\partial[y]_0/\partial S\right)}-\left.\frac{\partial\left(f(t,S)\cdot[\chi]_0\right)}{\partial[\chi]_0}\right|_{[\chi]_0=[\chi]_0^*,[\chi]_1=[\chi]_1^*}$$

The period of manufacturing macroscopic indices' distortion existence $T_{distortion}$ in practice amounts from several days up to several weeks, while the coefficients' $B_{\left(\partial[y]_1/\partial S\right)}$, $B_{\left(\partial[y]_0/\partial S\right)}$, $B_{([y]_1)}$, $B_{([y]_0)}$ change period is defined by the strategic operation of businesses and amounts from several months up to several years. The last statement gives an opportunity to consider input coefficients (22) evidently independent on time during the period of distortion existence $T_{distortion}$, as the values of coefficients $B_{\left(\partial[y]_1/\partial S\right)}$, $B_{\left(\partial[y]_0/\partial S\right)}$, $B_{([y]_1)}$, $B_{([y]_0)}$ change during the period of manufacturing macroscopic indices' distortion existence $T_{distortion}$ are significantly less than the values of the coefficients (22) themselves:

$$\frac{B_{\left(\partial[y]_1/\partial S\right)}}{T_{distortion}} >> \frac{\partial B_{\left(\partial[y]_1/\partial S\right)}}{\partial t}, \qquad \frac{B_{\left(\partial[y]_0/\partial S\right)}}{T_{distortion}} >> \frac{\partial B_{\left(\partial[y]_0/\partial S\right)}}{\partial t},$$

$$\frac{B_{([y]_1)}}{T_{distortion}} >> \frac{\partial B_{([y]_1)}}{\partial t}, \qquad \frac{B_{([y]_0)}}{T_{distortion}} >> \frac{\partial B_{([y]_0)}}{\partial t} \tag{23}$$

Thus we consider coefficients in the partial differential equations (21) dependent only on S. Let's take Fourier series of the macroparameters' $[\chi]_0$ and $[\chi]_1$ small distortions $[y]_0$, $[y]_1$:

$$[y]_0 = \{y_0\}_0 + \sum_{j=1}^{\infty}\{y_0\}_j \cdot \sin[k_j \cdot S] + \sum_{j=1}^{\infty}[y_0]_j \cdot \cos[k_j \cdot S]; \quad k_j = \frac{2\cdot\pi\cdot j}{S_d}$$

$$[y]_1 = \{y_1\}_0 + \sum_{j=1}^{\infty}\{y_1\}_j \cdot \sin[k_j \cdot S] + \sum_{j=1}^{\infty}[y_1]_j \cdot \cos[k_j \cdot S] \tag{24}$$

where $\{y_0\}_0$, $\{y_0\}_j$, $[y_0]_j$, $\{y_1\}_0$, $\{y_1\}_j$, $[y_1]_j$ are the coefficients of expansion in series the manufacturing system macroparameters' small distortions $[y]_0$, $[y]_1$ lengthwise the technological chain of the manufacturing process. Substituting instead $[y]_0$, $[y]_1$ their Fourier series development (24) in the equation (21), we obtain systems of equations for macroparameters' $[\chi]_0$, $[\chi]_1$ small distortions $[y]_0$, $[y]_1$ series expansion:

$$\frac{d\{y_0\}_0}{dt} = 0, \qquad \frac{d\{y_1\}_0}{dt} + B_{([y]_1)} \cdot \{y_1\}_0 + B_{([y]_0)} \cdot \{y_0\}_0 = 0, \tag{25}$$

and $$\frac{d\{y_0\}_j}{dt} - [y_1]_j \cdot k_j = 0, \qquad \frac{d[y_0]_j}{dt} + \{y_1\}_j \cdot k_j = 0 \tag{26}$$

$$\frac{d\{y_1\}_j}{dt} - B_{\left(\partial[y]_1/\partial S\right)} \cdot [y_1]_j \cdot k_j + B_{([y]_1)} \cdot \{y_1\}_j - B_{\left(\partial[y]_0/\partial S\right)} \cdot [y_0]_j \cdot k_j + B_{([y]_0)} \cdot \{y_0\}_j = 0$$

$$\frac{d[y_1]_j}{dt} + B_{\left(\partial[y]_1/\partial S\right)} \cdot \{y_1\}_j \cdot k_j + B_{([y]_1)} \cdot [y_1]_j + B_{\left(\partial[y]_0/\partial S\right)} \cdot \{y_0\}_j \cdot k_j + B_{([y]_0)} \cdot [y_0]_j = 0$$

with the corresponding characteristic equations

$$\begin{vmatrix} (\vartheta_0) & 0 \\ \left(B_{([y]_0)}\right) & \left(B_{([y]_1)} + \vartheta_0\right) \end{vmatrix} = 0, \qquad (j=0) \tag{27}$$

$$\begin{vmatrix} (\vartheta_j) & 0 & 0 & (-k_j) \\ 0 & (\vartheta_j) & (k_j) & 0 \\ \left(B_{([y]_0)}\right) & \left(-B_{\left(\partial[y]_0/\partial s\right)}\cdot k_j\right) & \left(\vartheta_j + B_{([y]_1)}\right) & \left(-B_{\left(\partial[y]_1/\partial s\right)}\cdot k_j\right) \\ \left(B_{\left(\partial[y]_0/\partial s\right)}\cdot k_j\right) & \left(B_{([y]_0)}\right) & \left(B_{\left(\partial[y]_1/\partial s\right)}\cdot k_j\right) & \left(\vartheta_j + B_{([y]_1)}\right) \end{vmatrix} = 0, \quad (j>1) \quad (28)$$

Characteristic equations (27) and (28) show the connection between the proper number of the characteristic equation ϑ_j and the wave number k_j:

$$\vartheta_0 \cdot \left(B_{([y]_1)} + \vartheta_0\right) = 0 \qquad \text{for j=0} \qquad (29)$$

$$\vartheta_j^2 + \vartheta_j \cdot \left(B_{([y]_1)} \pm i \cdot B_{\left(\partial[y]_1/\partial s\right)} \cdot k_j\right) + \left(k_j^{\,2} \cdot B_{\left(\partial[y]_0/\partial s\right)} \mp i \cdot k_j \cdot B_{([y]_0)}\right) = 0 \quad \text{for j>0.} \quad (30)$$

If the roots ϑ_j of the equations (29), (30) have negative real part, the manufacturing process is stable. The case of positive real part of ϑ_j witnesses of the exponential growth of distortions' $[y]_0$, $[y]_1$ amplitude in time, that is of instability. The system of equations of the manufacturing system state (25) has the characteristic equation (27) with one zero root $\vartheta_{0_1} \equiv 0$. In the stability theory such systems usually are related to the critical cases of motion stability research and require additional attention. The system of equations (25) has the following solution with respect to the small distortions $[y]_0$, $[y]_1$:

$$\{y_0\}_0 = c_{\{y_0\}_0} = \text{const}, \qquad \{y_1\}_0 = \exp\left(-B_{([y]_1)} \cdot t\right) + \{\tilde{y}_1\}_0 \qquad (31)$$

The constant of integration $\{\tilde{y}_1\}_0 = \text{const}$ can be defined from the equality:

$$B_{([y]_1)} \cdot \{\tilde{y}_1\}_0 + B_{([y]_0)} \cdot c_{\{y_0\}_0} = 0 \qquad (32)$$

The trivial solution $\{y_0\}_0 = c_{\{y_0\}_0} = 0$, $\{y_1\}_0 = 0$ contains in the family in the system under consideration solutions and corresponds to the zero value of constant $c_{\{y_0\}_0} = 0$. In this case the system of equations of manufacturing system state relatively to the small distortions $[y]_0$, $[y]_1$ supposes integral – the family of invariant surfaces, on each of which there is the particular point $\{y_0\}_0 = c_{\{y_0\}_0}$, $\{y_1\}_0 = \exp\left(-B_{([y]_1)} \cdot t\right) + \{\tilde{y}_1\}_0$. The manufacturing system's under consideration researched undisturbed stable state (31) corresponds to the trivial solution. In the same way other stable states of the manufacturing system under consideration correspond to the solution (31). So, in the particular case of one zero root, the investigated undisturbed state belongs to the family of the established states, which are defined by the system of equations (31). In the particular case the undisturbed state is always stable. The stability in this case is not asymptotical.

3. Conclusions. Thus, in the zero approximation on the small parameter Kv the type the base product distribution function on the expenses change rates $\chi(t,S,\mu)$ for description of the manufacturing system function is defined by means of the equation (10). The moments of the distribution function $\chi(t,S,\mu)$ comply to the closed system of equations (13), which helps to describe the macroscopic parameters of a perfect manufacturing system conduct. Perfectness of the manufacturing system is expressed by the absence of the elements, describing dissipative manufacturing process, for the closed system of equations (13) in the zero approximation on the small parameter Kv. Manufacturing system's technological process macroparameters' $[\chi]_0$ and $[\chi]_1$ stability conditions relatively to the small distortions $[y]_0$, $[y]_1$ can be written down like the negative real part of the characteristic equations (29) and (30). The obtained manufacturing operation macroparameters' stability conditions, in terms of system of equations (21) coefficients, give the relation between the value of the operational surpluses and the blank's motion pace from operation to operation lengthwise the technological process. The technological process operation macroparameters' $[\chi]_0$ and $[\chi]_1$ stability conditions define the conditions of raw materials, component delivery synchronization by the adjoining organizations and the structural areas of the enterprise. At the same time, it is supposed, that the system of equations (21), that describe the state of the manufacturing system relatively to the macroparameters' $[\chi]_0$, $[\chi]_1$ small distortions $[y]_0$, $[y]_1$ is analytical in the region under investigation and the researched undisturbed state (19) lies in the mentioned region.

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