PARAMETRIC INSTABILITY OF THE FREE SURFACE OF NONLINEAR MAGNETIZABLE FLUID

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Wave motions of a magnetic fluid are of great interest in science and are the subject of intense research. A strong factor influencing the stability is the modulation of temporal parameters of the equilibrium or fluid flow. Deserves special attention the situation where a change of parameters of the system is periodic in nature and can appear a phenomenon of parametric resonance. Periodic modulation of parameters is of great interest in practice in view of the prevalence of periodic random factors (temperature fluctuations, mechanical vibrations, sound and electromagnetic fields).

Interest to the problem of the stability of surface of the fluid in alternating fields is also related to the fact that in many hydrodynamic systems that are unstable in the absence of modulation, dynamic stabilization of the equilibrium is possible by using of parametric action. Thus, with the help of specially given modulation can be effectively controlled hydrodynamic stability.

The first investigations of movements of the surface waves belong to Faraday [1]. He studied the structure of wave motions. The result depended on the composition and depth of the liquid, the oscillation amplitude and frequency. Particularly in 1831 he found experimentally, that by supporting the forced oscillation frequency equal to the half of natural frequency of surface waves, a phenomenon known today as parametric resonance appears.

The first theory of this phenomenon was developed by T. Benjamin and F. Ursel [2] and independently by N. N. Moiseev [3, 4]. They showed that the expression for the displacement of surface of an ideal fluid in the linear approximation is reduced to the Mathieu equation, and therefore resonance frequencies exist [5], in which the surface is unstable. The current state of the problem is described by D. V. Lyubimov in the monograph [6]. K. Kumar and L. S. Tackerman [7] considered the problem of stability of two immiscible Newtonian viscous fluids contained in a vessel and subjected to periodic fluctuations. The problem of the stability of the interface viscous liquids of any viscosity was finally solved by K. Kumar [8].

During the 1960s were created magnetic fluids (ferrofluids) [9], which are artificial stable colloidal suspension of single-domain magnetic particles in a liquid carrier. The properties of magnetic fluids are mostly determined by the thermal Brownian motion of suspended particles and the fact that each single-domain particle has a permanent magnetization. The main idea is the ability to directly control the position of the ferrofluid and control its behavior using a magnetic field. This justifies the extensive studies of magnetic fluids, their peculiar physical and chemical properties, their hydrodynamic and thermal convection behavior [10].

The first investigations of the stability of the free surface of ferrofluid in the magnetic field belong to R. Rosensweig [11]. He found that the homogeneous stationary magnetic field, applied perpendicular to a flat layer of magnetic fluid, is the cause of spontaneous formation on the surface of the ordered structure of the sharp peaks when the magnitude of the field exceeds a critical value (Rosensweig instability). Then M. P. Perry and T. B. Jones [12] showed that the instability of a plane layer of ferrofluid can be excited by the tangential to the free surface time-periodic magnetic field. The influence of the time-only oscillating tangential magnetic field on the isothermal layer of ferrofluid was also examined by A. Cebers [13]. He analyzed the stability of the free surface of almost inviscid semi-infinite magnetic fluid, which is subjected to such field.

H. W. Muller [14] showed that the standing waves on the surface of the magnetic fluid can be excited in the normal steady-state magnetic field at the vertical vibration of the container, taking into account the effects of viscous dissipation and finite depth of fluid. He found that the mechanism of parametric excitation may cause a delay of Rosensweig instability. Also was examined the situation where a wave of Faraday and Rosensweig interact. V. Mekhonoshin and A. Lange [15] made a linear stability analysis of unlimited horizontal layer of magnetic fluid of arbitrary depth, which is subjected to vertical vibrations in a horizontal applied stationary magnetic field. M. Hennenberg, S. Slavtchev and B. Weyssow [16] have continued to develop the formulation of the linear stability problem for an isothermal layer of magnetic fluid exposed to a magnetic field, which contains a constant and an oscillating part. Two important cases were considered: the first one corresponding to the almost inviscid fluid, the second one taking the validity of the lubrication approximation.

In this paper we study the influence of mechanical vibration, time-dependent magnetic field and temperature fluctuations on the stability of the free surface of an ideal nonlinear magnetizable fluid. Thus, this problem is a generalization of the stability problem of the free surface in a constant magnetic field [11, 14, 15] to the case of unsteady arbitrarily oriented homogeneous field and most general isotropic law of magnetization. Also this problem is a generalization of problems [12, 13, 16] to the case of non-isothermal fluid flows.

1. Problem formulation. Let us consider a horizontal ferrofluid layer of infinite lateral extent and width (2), on top of which is located a nonmagnetic medium of lower density, such as air (1) (fig.1). Ferrofluid is considered ideal, non-conducting, incompressible and homogeneous. It is assumed that the nonlinear magnetizable fluid is in arbitrarily oriented to its free surface homogeneous time-varying magnetic field. We consider the case of the most general isotropic law of the magnetization.

It is assumed that $z = \zeta\,(x, y, t)$ is the interface between two semi-infinite fluid layers, and in a state of relative balance $z = 0$ is the equation of equilibrium of the interface.

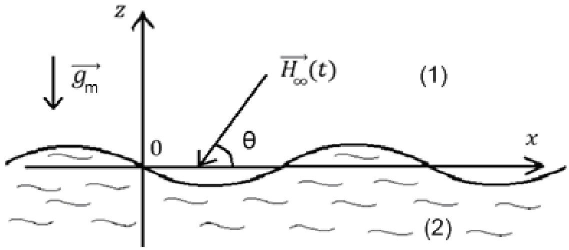

Fig.1. Schema of the problem under consideration. $z>0$: $\mu_\infty^{(1)}=1$,
$z<0$: $\mu_\infty^{(2)}=\mu(\rho,T,H)$

$\overrightarrow{g_m} = \left(-g+\omega_e^2 a_e\cos(\omega_e t)\right)\overrightarrow{e_z}$ modulation of the gravitational acceleration, a_e and ω_e – amplitude and frequency of oscillation of the surface $z = 0$ in the absolute coordinate system; θ – angle of orientation of magnetic field strength.

For regions occupied by the air and the ideal nonlinear magnetizable fluid we need to find the solution of equations:

$$\mathrm{div}\vec{V}=0 \tag{1}$$

$$\rho\left[\frac{\partial\vec{V}}{\partial t}+\left(\vec{V}\vec{\nabla}\right)\vec{V}\right]=-\nabla\left(p^0+\psi^{(p)}\right)+\rho\vec{g}_m+\vec{M}\nabla H$$

$$\frac{dS}{dt}=0;\quad S=S_0(T)+\frac{1}{\rho}\int_0^H M_T dH=\mathrm{const}$$

$$\mathrm{div}\vec{H}=-4\pi\,\mathrm{div}\left(\frac{M}{H}\vec{H}\right);\quad \mathrm{rot}\vec{H}=0;$$

$$M=M(\rho,T,H);\quad \psi^{(\rho)}=\int_0^H\left(M-\rho M_\rho\right)dH$$

The boundary conditions for the problem of parametric stability of the free surface include:

1. kinematic and dynamic conditions

$$z=\zeta(x,y,t):\qquad \langle V_n\rangle=0 \tag{2}$$

$$\langle p_0+\psi^{(p)}+2\pi M_n^2\rangle=-\alpha\,\mathrm{div}\vec{n}$$

2. conditions for the electromagnetic field

$$z=\zeta(x,y,t):\qquad \langle B_n\rangle=0;\langle\vec{H}_\tau\rangle=0 \tag{3}$$

3. boundary conditions at infinity

$$|z|\to\infty:\qquad \vec{V}^{(i)}=0 \tag{4}$$

$$\vec{H}^{(i)}=\left(H_{0x}(t),H_{0y}(t),H_{oz}/\mu_\infty^{(i)}\right)=\vec{H}_\infty^{(i)}$$

where $<a>=a_2-a_1$ is a jump of the corresponding quantities; i – index of the medium (1 – for air , 2 – for magnetizable fluid).
$\vec{n}$ – the normal to the free surface, which is directed into the medium at index 2.

Quantities entering into (1) – (4) mean:

$\vec{V}$ – velocity, ρ – density, p^0 – mechanical pressure, M – magnetization, $\psi^{(\rho)}$ – magnetostrictive pressure, $\vec{H}$ – magnetic field strength, S – entropy, T – temperature, α – the surface tension coefficient.

1.1. Generalization of the integral of Cauchy–Lagrange to the case of accounting in magnetizable fluid magnetocaloric effect. We assume that the motion of the fluid emerges from dormancy. The system of equations for our problem has two integrals.

First, for the most general law of the magnetization, taking into account the magnetocaloric effect, the motion is not isothermal but adiabatic, so that in each region:

$$S_0(T)+\frac{1}{\rho}\int_0^H M_T dH=\text{const}; \quad S_0(T)=c_V\ln(T)+\text{const} \tag{5}$$

where c_V is a volume heat capacity.

This integral allows to determine the temperature as a function of magnetic field strength:

$$T^{(i)}=T^{(i)}\left(\rho,H^{(i)}\right),\ i=1,2$$

Therefore, the magnetization in each region will be a function of magnetic field strength:

$$M^{(i)}=M^{(i)}\left(\rho,T^{(i)}\left(H^i,\rho\right),H^{(i)}\right)=\tilde{M}^{(i)}\left(H^{(i)}\right),\ i=1,2$$

Thus, the ponderomotive force

$$M\nabla H=\nabla\int_0^H\tilde{M}dH$$

is potential, so that if the motion occurs from rest the conditions of Lagrange's theorem are satisfied and the resulting motion will be irrotational:

$$\text{rot}\vec{V}=0\Rightarrow\vec{V}=\nabla\varphi$$

where φ is the velocity potential. It follows that the equation of motion admits the integral of the Lagrange–Cauchy:

$$\rho\varphi_t+\frac{\rho}{2}(\nabla\varphi)^2+\rho g_m z+p_0+\psi^{(\rho)}-\int_0^H\tilde{M}dH=c(t) \tag{6}$$

1.2. Formulation of problem in terms of potentials. Note that in view of equation

$$\text{rot}\vec{H}=0$$

the magnetic field strength can be represented as

$$\vec{H} = \nabla\Phi$$

where Φ is the magnetic field potential.

Formulation of the problem in terms of potential of velocity and potential of the magnetic field strength:

$$\Delta\phi = 0 \tag{7}$$

$$\Delta\Phi = -4\pi\mathrm{div}\left[\frac{M(\rho, T(|\nabla\Phi|), |\nabla\Phi|}{|\nabla\Phi|}\nabla\Phi\right]$$

$z = \zeta(x, y, t)$:

$$< 2\pi M_n^2 + \int \tilde{M}dH - \rho\phi_t - \frac{\rho}{2}(\phi_x^2 + \phi_y^2 + \phi_z^2) > -g_m\zeta < \rho > = -\alpha\mathrm{div}\vec{n}$$

$$\zeta_t = \varphi_z^{(1)} - \varphi_x^{(1)}\zeta_x - \zeta_y\varphi_y^{(1)} = \varphi_z^{(2)} - \varphi_x^{(2)}\zeta_x - \zeta_y\varphi_y^2$$

$$< \mu\Phi_t > = \zeta_x < \mu\Phi_x > + \zeta_y < \mu\Phi_y >$$

$$< \Phi_x > + \zeta_x < \Phi_z > = 0, < \Phi_y > + \zeta_y < \Phi_z > = 0$$

$|z| \to \infty$:

$$\nabla\phi^{(i)} \|_{|z|\to\infty} \to 0; \quad \nabla\Phi^{(i)} \|_{|z|\to\infty} = \vec{H}_\infty^{(i)}, \ i = 1,2$$

where μ is permeability.

This formulation of the problem was used by I. E. Tarapov for investigation the problem of instability of the fluids interface in a stationary field [17]. We generalize this problem to the case of time-dependent magnetic field and take into account magnetocaloric effect.

1.3. Linearization of the problem. Denote $\phi' = \phi - \phi_0$ is a perturbation of the velocity potential, $\overrightarrow{H'} = \nabla\Phi' = \nabla\Phi - \vec{H}_\infty$ – perturbation of the magnetic field strength. To investigate the stability of the horizontal surface z = 0 we linearize our problem by assuming

$$\frac{k}{2\pi}|\zeta|, |\zeta_x|, |\zeta_y|, \frac{|\overrightarrow{H'}|}{|\vec{H}_\infty|}, \frac{|\nabla\phi'|^2}{|\phi_t'|} \sim \varepsilon << 1$$

Thus, we neglect quantities of the order $O(\varepsilon^2)$.

The linearized problem for the potentials φ and Φ has the form (the tilde over the perturbed quantities is omitted):

$$z > 0: \qquad \Delta\phi^{(1)} = 0, \qquad \Delta\Phi^{(1)} = 0 \tag{8}$$

$$z < 0: \qquad \Delta\phi = 0, \qquad \Delta\Phi = -c_\infty\vec{H}_\infty\nabla(\vec{H}_\infty\nabla\Phi)$$

$$z = 0: \qquad \zeta_t = \varphi_z$$

$$\rho\phi_t + \rho g_m\zeta - \alpha(\zeta_{xx} + \zeta_{yy}) = \frac{1}{4\pi}\left\{\frac{(\mu-1)^2}{\mu}H_{0z}(\Phi_z - \vec{H}_\infty\nabla\zeta) + \frac{(\mu-1)(\mu + c_\infty H_{0z}^2)}{\mu}(\vec{H}_\infty\nabla\Phi)\right\}$$

$$\mu\Phi_z - \Phi_z^{(1)} + c_\infty H_{0z}(\vec{H}_\infty\nabla\Phi) = (\vec{H}_\infty\nabla\zeta)(\mu - 1)$$

$$\Phi_x - \Phi_x^{(1)} + H_{0z}\zeta_x\left(\frac{1}{\mu}-1\right)=0$$

$$\Phi_y - \Phi_y^{(1)} + H_{0z}\zeta_y\left(\frac{1}{\mu}-1\right)=0$$

$|z| \to\infty$: $\nabla\varphi\,|_{|z|\to\infty}\to 0;\ \nabla\Phi\,|_{|z|\to\infty}=0$

Where $\phi \equiv \phi^{(2)},\ \Phi \equiv \Phi^{(2)}, c_\infty = \dfrac{4\pi}{\mu_\infty H_\infty^2}\left\{\left(\dfrac{\partial M}{\partial H}\right)_\infty - \dfrac{T_\infty}{c_V\rho}\left(\dfrac{\partial M}{\partial T}\right)_\infty^2 - \left(\dfrac{M}{H}\right)_\infty\right\}$

The formulated problem allows to investigate the parametric instability of the free surface of magnetic fluid in the case of the nonlinear dependence of the magnetization on the field strength and temperature.

2. Solution of the problem. The solution of problem (8) is sought in the form:

$$\zeta(t,x,y)=a(t)e^{i(\vec{k}\,\vec{r})};\ \vec{k}=(k_x,k_y),\ \vec{r}=(x,y); \tag{9}$$

$z>0$: $\phi^{(1)}=0,\qquad \Phi^{(1)}(t,x,y,z)=c^{(1)}(t)\psi^{(1)}(z)e^{i(\vec{k}\,\vec{r})}$

$z<0$: $\phi(t,x,y,z)=b(t)\theta(z)e^{i(\vec{k}\,\vec{r})},\quad \Phi(t,x,y,z)=c(t)\psi(z)e^{i(\vec{k}\,\vec{r})}$

where k is the wave number of the perturbations arising on the free surface.

Thus, we neglect the influence of air on the fluid motion.

The problem is reduced to the investigation of the equation for the amplitude of a perturbation of the free surface of a magnetizable fluid.

$$\ddot{a}+2\gamma\dot{a}+a\left\{kg_m+\frac{\alpha k^3}{\rho}-\frac{k^2(\mu-1)^2}{4\pi\rho\mu}\left[\frac{H_{0z}^2\sqrt{1+c_{1\infty}}-H_\tau^2\mu}{1+\mu\sqrt{1+c_{1\infty}}}\right]\right\}=0 \tag{10}$$

Where $c_{1\infty}=\dfrac{4\pi}{\mu_\infty}\left\{\left(\dfrac{\partial M}{\partial H}\right)_\infty-\dfrac{T_\infty}{c_V\rho}\left(\dfrac{\partial M}{\partial T}\right)_\infty^2-\left(\dfrac{M}{H}\right)_\infty\right\},\qquad H_\tau=\dfrac{(\vec{H}_\infty\vec{k})}{k}$

Attenuation of surface waves is taken into account by introducing into the equation dissipation coefficient in the form, obtained in [7]:

$$\gamma=2k^2\frac{\eta}{\rho} \tag{11}$$

2.1. Study of the equation. Let θ is the angle of orientation of the magnetic field strength. Then:

$$H_{\infty z}=H_\infty\sin\theta,\quad H_{\infty\tau}=H_\infty\cos\theta \tag{12}$$

and the equation (10) can be written in the form:

$$\ddot{a}+2\gamma\dot{a}+a\left\{kg_m+\frac{\alpha k^3}{\rho}-\frac{k^2(\mu_\infty-1)^2}{4\pi\rho}H_\infty^2\left[\sin^2\theta-\frac{1}{1+\mu_\infty\sqrt{1+c_{1\infty}}}\right]\right\}=0 \tag{13}$$

The equation of oscillations of a dissipative system with one degree of freedom is reduced to:

$$\ddot{a} + 2\gamma \dot{a} + a F(t) = 0 \tag{14}$$

where the function F(t) in our case has the form:

$$F(t) = k g_m(t) + \frac{\alpha k^3}{\rho} - \frac{k^2(\mu_\infty - 1)^2}{4\pi\rho} H_\infty^2(t)\left[\sin^2\theta - \frac{1}{1 + \mu_\infty\sqrt{1 + c_{1\infty}}}\right]$$

In what follows we consider the case of a periodic function F(t) with period *T*. When $\gamma = 0$ from equation (14) we obtain the Mathieu–Hill's equation:

$$\ddot{a} + a F(t) = 0 \tag{15}$$

If $\gamma \neq 0$, then equation (14) reduces to (15) by substitution

$$a(t) = e^{-\gamma t} u(t)$$

If the function F(t) is piecewise–constant, then equation (15) is called the Meissner equation.

Particular interest for applications is the case of harmonic excitation, corresponding to the instructions of the following relationships:

$$H_\infty(t) = H_{00}\left(1 + \frac{h_\tau}{H_{00}}\cos\omega_\tau t + \frac{h_n}{H_{00}}\cos\omega_n t\right),\ g_m = g\left(1 - \frac{\omega_e^2 a_e}{g}\cos\omega_e t\right) \tag{16}$$

where h_τ and $\omega_\tau -$ amplitude and frequency of oscillation of the tangential component of magnetic field strength, h_n and $\omega_n -$ amplitude and frequency of oscillation of the normal component of magnetic field strength, a_e and $\omega_e -$ amplitude and frequency of oscillation of modulation of the gravitational acceleration.

Remark that since the amplitudes h_τ and h_n can be chosen as follows:

$$h_\tau = O(\varepsilon^m),\ h_\tau = O(\varepsilon^m),\ m < 1/2$$

as they are defined by specifying the external field.

Then the function F(t) has the form:

$$F(t) = 1 + q_1\cos\omega_e t + q_2\cos\omega_\tau t + q_1\cos\omega_n t,$$

where $1 = kg + \dfrac{\alpha k^3}{\rho} - \dfrac{k^2(\mu_\infty - 1)^2}{4\pi\rho} H_{00}^2\left[\sin^2\theta - \dfrac{1}{1 + \mu_\infty\sqrt{1 + c_{1\infty}}}\right]$, $q_1 = -k\omega_e^2 a_e$,

$$q_2 = -\frac{k^2(\mu_\infty - 1)^2}{4\pi\rho}\left[\sin^2\theta - \frac{1}{1 + \mu_\infty\sqrt{1 + c_{1\infty}}}\right] h_\tau^2$$

$$q_3 = -\frac{k^2(\mu_\infty - 1)^2}{4\pi\rho}\left[\sin^2\theta - \frac{1}{1 + \mu_\infty\sqrt{1 + c_{1\infty}}}\right] h_n^2$$

Thus, for harmonic excitation the equation (15) is a Mathieu equation:

$$\ddot{a}+a\left(1+q_1\cos\omega_e t+q_2\cos\omega_\tau t+q_1\cos\omega_n t\right)=0, \qquad (17)$$

if the frequency ratio ω_e,ω_τ and ω_n is a rational number.

In what follows we consider the case: $\omega_e=\omega_\tau=\omega_n$. Then (17) takes the following form:

$$\ddot{a}+a\left(1+q\cos\omega t\right)=0, \qquad (18)$$

where $$1=kg+\frac{\alpha k^3}{\rho}-\frac{k^2(\mu_\infty-1)^2}{4\pi\rho}H_{00}^2\left[\sin^2\theta-\frac{1}{1+\mu_\infty\sqrt{1+c_{1\infty}}}\right],$$

$$q=-k\omega_e^2 a-\frac{k^2(\mu_\infty-1)^2}{4\pi\rho}\left[\sin^2\theta-\frac{1}{1+\mu_\infty\sqrt{1+c_{1\infty}}}\right]\left(h_\tau^2+h_n^2\right)$$

As follows from the above studies, the stability of the free surface of an ideal magnetizable fluid is described by Mathieu equation. It depends on two coefficients *q* and *l*, which fully determine the stability of motion.

The plane of changes of *q* and *l* can be divided into regions corresponding to stable and unstable motions (Ince–Strutt diagram).

3. The case of a constant magnetic field. In this case from the equation (10) follows known result [11, 17] about the stability of the surface in a stationary magnetic field. The equation for perturbations of the free surface of ideal magnetizable fluid has the form:

$$\ddot{a}+\omega^2(k)a=0, \qquad (19)$$

where $$\omega^2(k)=\frac{\alpha k^3}{\rho}-\frac{k^2(\mu-1)^2}{4\pi\rho(1+\mu)}\left[\mu H_{\infty z}^2-H_{\infty\tau}^2\right]+kg$$

The condition for instability of the solutions of (19) (existence of real values of wave number k) consists in the following inequality:

$$\omega^2(k)<0 \qquad (20)$$

Hence it follows that the instability of the free surface occurs if the vertical component of magnetic field strength $H_{\infty z}$ exceeds the critical value

$$H_{\infty z}^2>H_R^2=\frac{H_{\infty\tau}^2}{\mu}+\frac{4\pi}{(\mu-1)^2}\left(1-\frac{1}{1+\mu}\right)\left(\alpha k+\frac{\rho g}{k}\right) \qquad (21)$$

The most dangerous perturbations are those with the length of the wave vector

$$k_R=\sqrt{\frac{\rho g}{\alpha}} \qquad (22)$$

Thus, when

$$H_{\infty z}^2>H_R^2=\frac{H_{\infty\tau}^2}{\mu}+\frac{8\pi\sqrt{\rho g\alpha}}{(\mu-1)^2}\left(1-\frac{1}{1+\mu}\right) \qquad (23)$$

the instability occurs, which is called the Rosensweig's instability. In this case the horizontal component of the magnetic field strength $H_{\infty\tau}$ has a stabilizing effect on the surface.

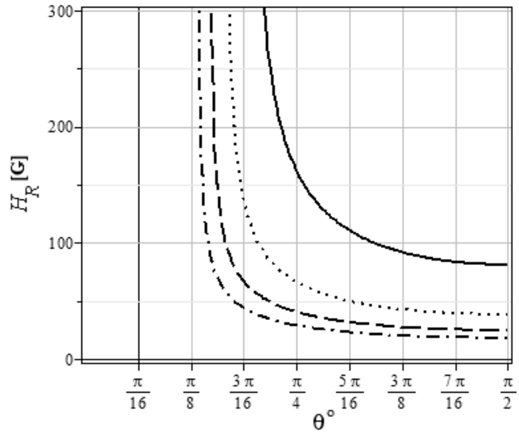

Fig.2. The dependence of the critical value of magnetic field strength H_R from the angle θ (orientation of the field) for different values of permeability: $\mu = 2$ (solid line), $\mu = 3$ (dotted line), $\mu = 4$ (dashed line), $\mu = 5$ (dashed-dotted line).

Using (12) the condition for instability (20) takes the following form:

$$\omega^2(k) = \frac{\alpha k^3}{\rho} - \frac{k^2(\mu-1)^2}{4\pi\rho} H_\infty^2 \left[\sin^2\theta - \frac{1}{1+\mu}\right] + kg < 0 \qquad (24)$$

Fig.2 shows that with increasing a horizontal component of magnetic field strength, possible to move further threshold for the onset of Rosensweig's instability. If the orientation of the magnetic field is such, that does not satisfy (24), then the instability does not appear.

3.1. The case of combined impact of mechanical vibrations and a constant magnetic field. Let $H_\infty = \text{const}$, $g_m = g\left(1 - \frac{\omega_e^2 a_e}{g}\cos\omega_e t\right)$. Then the equation for the perturbation of the free surface of an ideal magnetizable fluid reduces to Mathieu equation (18), which by substitution $t \to \omega_e t/2$ takes the form:

$$\ddot{a} + (\delta + 2\varepsilon\cos 2t) = 0 \qquad (25)$$

$$\delta = \frac{4}{\omega_e^2}\left(4k^4\frac{\eta^2}{\rho^2} + \frac{\alpha k^3}{\rho} - \frac{k^2(\mu-1)^2}{4\pi\rho} H_\infty^2\left[\sin^2\theta - \frac{1}{1+\mu}\right] + kg\right), \quad \varepsilon = -2ka_e$$

Let us analyze the influence of vibrations of the gravitational field on the stability of the free surface in a stationary magnetic field applied.

We considered the magnetic fluid is water based with typical values of parameters:

$\rho = 1{,}19\left(\dfrac{\mathrm{g}}{\mathrm{cm}^3}\right)$ – density; $\eta = 0{,}07(\mathrm{P})$ – viscosity; $\alpha = 26\left(\dfrac{\mathrm{dyn}}{\mathrm{cm}}\right)$ – the surface tension coefficient.

Fig.3 and Fig.4 show that when the vertical component of the magnetic field strength does not exceed the critical value $\mathrm{H_R}$, then on the free surface appear waves , the length of which depends on the amplitude and frequency of modulation of the gravitational acceleration. But when $\mathrm{H_z} > \mathrm{H_R}$ Rosensweig's instability occurs and length of the excited waves decreases sharply.

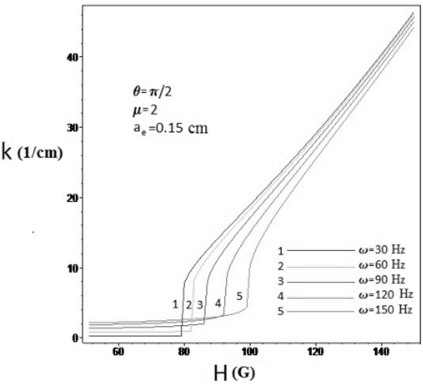

Fig.3. The dependence of the wave number of perturbations arising on the free surface of a magnetic fluid on the magnitude of the vertical magnetic field strength for different values of the modulation frequency of the gravitational field.

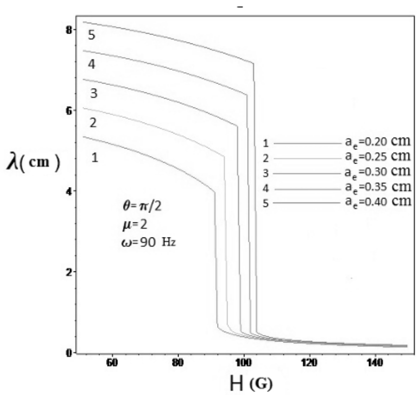

Fig.4. The dependence of the length of the excited waves on the magnitude of the vertical magnetic field strength for different values of the amplitude of gravitational modulation.

As well as in [14], Fig.3 and Fig.4 show that possible to move further threshold for the onset of Rosensweig's instability by using modulation of the gravitational acceleration. Make a conclusion, that when the frequency of modulation of the gravitational acceleration increases, the length of the excited waves on the free surface of a magnetic fluid decreases. And with the increasing of amplitude of the gravitational field, wavelength increases too.

Fig.5 shows, that the higher the permeability of the liquid is, the faster the Rosensweig's instability occurs. Therefore interesting is the case of the nonlinear dependence $\mu = \mu(\rho, \mathrm{T}, \mathrm{H})$. As well as in [15], Fig.6 demonstrates that the tangential component of the magnetic field strength exerts a stabilizing effect on the free surface of the ferrofluid, which is subjected to vertical vibrations.

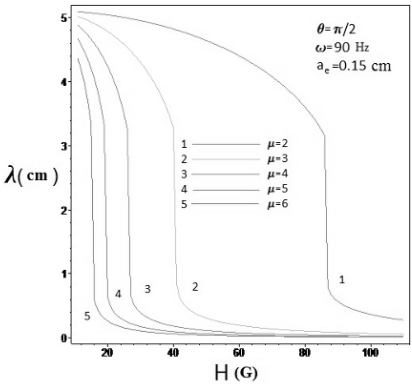

Fig.5. The dependence of the length of excited waves on the magnitude of the vertical magnetic field strength for different values of permeability.

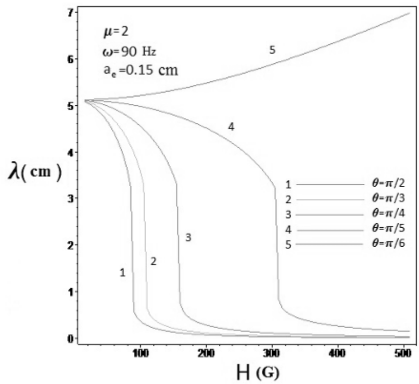

Fig.6. The dependence of the length of excited waves on the magnitude of the magnetic field strength depending on magnetic field orientation.

3.2. The case of non-stationary magnetic field.

3.2.1. Model of an ideal paramagnetic medium. In this case, the equation for the amplitude of the perturbation of the free surface of a magnetizable fluid has the form (13). As a model of an ideal paramagnetic medium choose the ferrofluid, magnetization of wich can be described by the Langevin's law of the magnetization:

$$M = nm\,L(\xi),\ L(\xi) = \mathrm{cth}\,\xi - \xi^{-1},\ \xi = \frac{mH}{k_b T}, \tag{26}$$

where n – volume concentration of ferroparticles; m – magnetic moment of a single particle; k_b – the Boltzmann constant;

We define the magnetic field strength in such way, that it contains a constant and oscillating parts:

$$H_\infty = H_{0\infty}\left(1 + \frac{h}{H_{0\infty}}\right),\ h = \varepsilon_h H_{0\infty} \cos \omega t, \tag{27}$$

where $\varepsilon_h H_{0\infty} -$ amplitude of parametric excitation

Then the following parameters can be represented as:

$$T = T_0 + T',\ \xi = \xi_0 + \xi',\ \mu = \mu_0 + \mu'$$

where the coefficients with primes denote perturbations of the corresponding quantities.

From the integral of adiabaticity (5):

$$S = c_v \ln T + \frac{n k_b}{\rho}\left[\ln\frac{\mathrm{sh}\,\xi}{\xi} - \xi L(\xi)\right] = \mathrm{const} \tag{28}$$

find the connection between the perturbations of temperature and magnetic field strength:

$$T' = \frac{T_0 n k_b}{c_v}\frac{\xi_0}{H_{0\infty}}\frac{\left(-\frac{1}{\mathrm{sh}^2\xi_0} + \frac{1}{\xi_0{}^2}\right)}{1 + \frac{n k_b}{c_v}\xi_0{}^2\left(-\frac{1}{\mathrm{sh}^2\xi_0} + \frac{1}{\xi_0{}^2}\right)} h \tag{29}$$

For the model of an ideal paramagnetic medium magnetocaloric effect is insignificant. From the expression (21) we get:

$$T_0(H) = T_{00}\left(1 - \frac{n k_b}{c_v}\left[\ln\frac{\mathrm{sh}\,\xi_0}{\xi_0} - \xi_0 L(\xi_0)\right]\right), \tag{30}$$

where $T_{00} -$ fluid temperature in the absence of a magnetic field.

For typical parameters of water-based ferrofluid, even with a significant increase of the magnetic field strength, the fluid temperature varies very little.

If the magnetic field strength is given in the form (27), equation (13) reduces to the Mathieu equation (25) with coefficients:

$$\delta = \frac{4}{\omega_e^2}\left(-4k^4\frac{\eta^2}{\rho^2} + \frac{\alpha k^3}{\rho} - \frac{k^2}{4\pi\rho}P_0 - gk\right),$$

$$\varepsilon = -\frac{2}{\omega_e^2}\frac{k^2}{4\pi\rho}P'\frac{\xi_0}{1 + \frac{n k_b}{c_v}\xi_0{}^2\left(-\frac{1}{\mathrm{sh}^2\xi_0} + \frac{1}{\xi_0{}^2}\right)}\varepsilon_h\ ,$$

$$\mu_\infty^0 = 1 + \frac{4\pi m c\rho L(\xi_0)}{H_{0\infty}},\ P_0 = 4\pi nm\left(\mathrm{cth}\,\xi_0 - \xi_0{}^{-1}\right)\left[\sin^2\theta - \frac{1}{1 + \mu_\infty^0\sqrt{1 + c_{1\infty}^0}}\right]$$

$$c_{1\infty}^0 = \frac{4\pi n m}{\mu_\infty^0 H_{0\infty}}\left\{\xi_0\left(-\frac{1}{\mathrm{sh}^2\xi_0} + \frac{1}{\xi_0{}^2}\right) - \frac{n k_b}{c_v}\xi_0{}^3\left(-\frac{1}{\mathrm{sh}^2\xi_0} + \frac{1}{\xi_0{}^2}\right)^2 - L(\xi_0)\right\}$$

$$P' = 2P_0 L(\xi_0)\left[\left(-\frac{1}{\mathrm{sh}^2\xi_0} + \frac{1}{\xi_0{}^2}\right) - \frac{1}{\xi_0}\left(L(\xi_0) + 2\right)\left(1 + \frac{n k_b}{c_v}\xi_0{}^2\left(-\frac{1}{\mathrm{sh}^2\xi_0} + \frac{1}{\xi_0{}^2}\right)\right)\right] -$$

$$-\frac{(\mu_\infty^0 - 1)^2 H_{0\infty}^2}{(1 + \mu_\infty^0\sqrt{1 + c_{1\infty}^0}}\left\{\frac{\mu_\infty^0 c_{1\infty}'}{2\sqrt{1 + c_{1\infty}^0}} + \sqrt{1 + c_{1\infty}^0}\,\frac{4\pi n m}{H_{0\infty}}\left[\left(-\frac{1}{\mathrm{sh}^2\xi_0} + \frac{1}{\xi_0{}^2}\right) -\right.\right.$$

$$
\left.-\frac{L(\xi_0)}{\xi_0}\left(1+\frac{n\,k_b}{c_v}\xi_0{}^2\left(-\frac{1}{sh^2\xi_0}+\frac{1}{\xi_0{}^2}\right)\right)\right\}=0
$$

$$
c'_{1\infty}=c^0_{1\infty}\left(-\frac{4\pi\,n\,m}{H_{0\infty}}\left[\left(-\frac{1}{sh^2\xi_0}+\frac{1}{\xi_0{}^2}\right)-\frac{1}{\xi_0}\left(L(\xi_0)+2\right)\left(1+\frac{n\,k_b}{c_v}\xi_0{}^2\left(-\frac{1}{sh^2\xi_0}+\frac{1}{\xi_0{}^2}\right)\right)\right]-\right.
$$

$$
-\frac{1}{\xi_0}\left(1+\frac{n\,k_b}{c_v}\xi_0{}^2\left(-\frac{1}{sh^2\xi_0}+\frac{1}{\xi_0{}^2}\right)\right)+\frac{4\pi\,n\,m}{\mu^0_\infty H_{0\infty}}\left\{\xi_0\left(-\frac{1}{sh^2\xi_0}+\frac{1}{\xi_0{}^2}\right)-\right.
$$

$$
\left.\left.-\frac{n\,k_b}{c_v}\left(2\xi_0{}^3\left(-\frac{1}{sh^2\xi_0}+\frac{1}{\xi_0{}^2}\right)\left(\frac{2ch\xi_0}{sh^3\xi_0}-\frac{2}{\xi_0{}^2}\right)+3\xi_0{}^2\left(-\frac{1}{sh^2\xi_0}+\frac{1}{\xi_0{}^2}\right)^2\right)\right\}\right)
$$

Fig.9 shows that when the vertical component of the magnetic field strength does not exceed the critical value H_R, then on the free surface appear waves , the length of which depends on the amplitude and frequency of oscillating part of magnetic field. But when $H_z > H_R$ Rosensweig's instability occurs and length of the excited waves decreases sharply. Fig.9 shows, that by using an oscillating magnetic field possible to move the threshold for the onset of Rosensweig's instability.

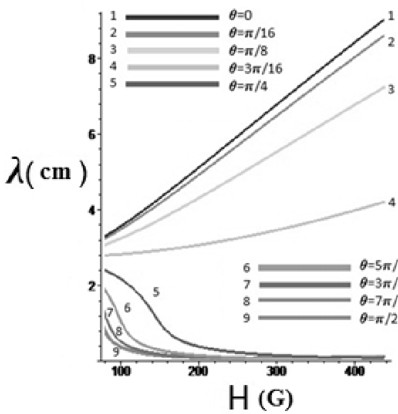

Fig.7. The dependence of the length of the excited waves on the magnitude of magnetic field strength at different orientations of the magnetic field relative to the free surface of the ferrofluid.

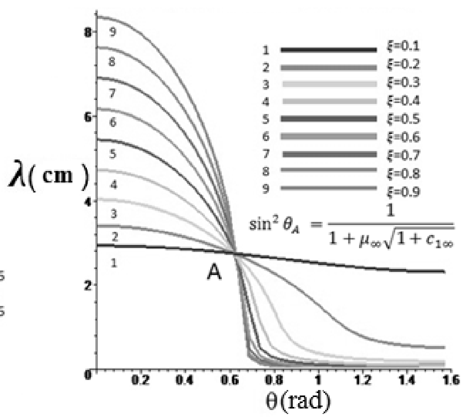

Fig.8. The dependence of the length of the excited waves on the orientation of the magnetic field for different values of the Langevin's parameter.

$$n = 1.11910^{17}\left(cm^{-3}\right);\ c_V = 4{,}18410^{7}\left(\frac{erg}{cm^3 K}\right);\ m = 0{,}8610^{-16}\left(\frac{dyn\,cm}{G}\right);$$

$$\eta = 0{,}07(P);\ \alpha = 26\left(\frac{dyn}{cm}\right);\ \rho = 1{,}19\left(\frac{g}{cm^3}\right);$$

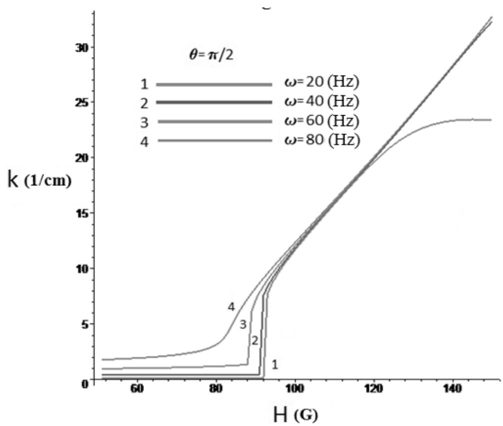

Fig.9. The dependence of the wave number of perturbations that appear on the free surface of a magnetic fluid on the magnitude of the vertical component of magnetic field strength at different values of the frequency of oscillating part of magnetic field.

3.2.2. Model of a nonideal paramagnetic medium. Let us consider the following law of the magnetization:

$$M = \rho\kappa\left(T - T_0\right) \tag{31}$$

where κ is a pyromagnetic coefficient. We define the temperature in such way, that it contains a constant and oscillating parts:

$$T = T_c + T',\ \ T' = \varepsilon_T T_c \cos\omega t, \tag{32}$$

where $\varepsilon_T T_c$ is the amplitude of parametric excitation.

From the integral of adiabaticity (5) we find the connection between the perturbations of temperature and magnetic field strength:

$$T' = -\frac{T_c k}{c_V} h \tag{33}$$

As a result of temperature fluctuations, oscillations of magnetic field strength will occur and we get the case of parametric excitation of waves, which is observed in the previous section.

If the temperature is given in the form (32), equation (13) reduces to the Mathieu equation (25) with coefficients:

$$\delta=\frac{4}{\omega^2}\left(-4k^4\frac{\eta^2}{\rho^2}+\frac{\alpha k^3}{\rho}-\frac{k^2}{4\pi\rho}\left[4\pi\rho\kappa\left(T_c-T_0\right)\right]^2\left\{\sin^2\theta-\frac{1}{1+\mu_c\sqrt{C_{1\infty}^0}}\right\}+kg\right)$$

$$\mu_c=1+4\pi\rho\kappa\frac{T_c-T_0}{H_c};\quad \varepsilon=\frac{4}{\omega^2}\left(-\frac{k^2}{4\pi\rho}\varepsilon_T\, T_c\left(4\pi\rho\kappa\right)^2\left(T_c-T_0\right)\left\{\sin^2\theta-\right.\right.$$

$$\left.\left.-\frac{1}{1+\mu_c\sqrt{1+C_{1\infty}^0}}\left(1-\frac{T_c-T_0}{2}\frac{C_{\mu'}\sqrt{1+C_{1\infty}^0}+\frac{\mu_c C_{c'}}{2\sqrt{1+C_{1\infty}^0}}}{1+\mu_c\sqrt{1+C_{1\infty}^0}}\right)\right\}\right)$$

$$C_{c'}=\frac{4\pi\rho\kappa}{\mu_c}\left\{-\frac{C_{\mu'}}{\mu_c}\left(\frac{\rho k T_c}{c_v}-\frac{T_c-T_0}{H_c}\right)+\frac{\rho\kappa}{c_v}-\frac{1}{H_c}-\frac{\left(T_c-T_0\right)c_v}{H_c^2T_ck}\right\}$$

$$C_{1\infty}^0=\frac{4\pi\rho\kappa}{\mu_c}\left\{\frac{\rho k T_c}{c_v}-\frac{T_c-T_0}{H_c}\right\},\ C_{\mu'}=\frac{4\pi\rho\kappa}{\mu_c}\left(1+\frac{c_v(T_c-T_0)}{H_cT_ck}\right),$$

Thus, we can see the possibility of excitation of parametric instability of the free surface of a magnetizable fluid as a result of harmonic perturbation of its temperature due to the magnetocaloric effect, which becomes important at phase transitions (at the Curie temperature or at structuring of the magnetic fluid).

4. Conclusions. In this paper we obtained an equation for the amplitude of the perturbation of free surface of nonlinear magnetizable fluid for multifrequency parametric excitation (modulation of the gravitational acceleration, different frequencies of the normal and tangential to the equilibrium surface components of the magnetic field strength, temperature fluctuations).

The dependence of the lengths of the excited waves from the angle of the magnetic field orientation at various values of field strength and frequency of magnetic field perturbations is discussed for the first time. It is shown the principle possibility of excitation of parametric instability of the free surface of a magnetizable liquid as a result of harmonic perturbation of the temperature for a nonideal paramagnet due to the magnetocaloric effect.

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